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DRAFT: FLUID STRUCTURE INTERACTION ANALYSIS VIA FIXED-POINT METHOD AND GEOMETRICALLY EXACT APPROACH

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ABSTRACT
To study the fluid structure interaction with large structural deformation, a fixed point method with Aitken's dynamic relaxation is used to accelerate convergence of the coupling iteration, and geometrically exact approach proposed by Simo is adopted to simulate the geometrically nonlinear dynamics of flexible beams. In order to reduce the error of the computation of structural dynamics, an improved implicit time integration algorithm based on Simo-Newmark method is presented. The case of vortex induced vibration of a cantilever is computed to verify the applicability of the approach based on geometrically exact beam theory and the efficiency of the fixed-point method is discussed.

INTRODUCTION
Fluid structure interaction (FSI) problems involve the interaction between fluid forces and structural responses and are encountered in many engineering problems, especially in aeroelasticity. With the growth of interest in high-altitude, long-endurance (HALE) aircraft , new methods need to be developed to study the aeroelastic behavior of high aspect ratio wings. In the FSI computation of aeroelastic problems, there are two main kinds of coupling methods: the monolithic approach and the partition approach. Because of its flexibility of choosing different solvers, the partition method is favored in current computational aeroelastic researches. A possible partitioning strategy that enables the reuse of existing solvers is the Dirichlet-Neumann partitioning [1]. There are two ways of partition approach: loose coupling and strong coupling. Loose coupling is easy to implement and widely used in many literatures [2-4]. However, it generates an error at each time step. Farhat and Lesoinne [5] proposed two improved loose coupling algorithms with second-order accuracy in time. In most researches on aeroelasticity, the motion of structure is governed by linear modal equations. However, for the structures with large deformation, nonlinear dynamic equations should be used to describe their motion. To simulate the FSI coupling between fluid and flexible structure, a strong coupling method is called Block-Gauss-Seidel or fixed-point method can be employed. This method is widely used in researches on incompressible flow and structures with large deformation [6-9].

As for the computation of the geometrically nonlinear structure of high aspect ratio wings, the geometrically exact intrinsic beam model proposed by Hodges [10] is widely used in aeroelasticity. However, the shortcoming of this method is: as the number of discrete nodes increases, the number of independent variables increases exponentially, furthermore the set of equations become stiff and lead to low efficiency in numerical calculation. In this paper, the geometrically exact beam model proposed by Simo [11] is adopted to simulate the nonlinear structural dynamics. The organization of this paper is as follows: Section II describes the FSI coupling system, and discusses the fixed-point method with dynamic Aitken's relaxation. Section III gives an introduction to geometrically exact beam theory to study the geometrically nonlinear dynamics of slender structures. Section IV computes the problem of the vortex induced vibration of a flexible cantilever to verify the application of geometrically exact beam model in FSI simulation. And different coupling strategies are also compared.

DESCRIPTIONS OF COUPLING SYSTEM

To use the partition method, the FSI system could be regarded as a domain consisting of non-overlapping fluid and structural domains. A Dirichlet-to-Neumann nonlinear operator associating fluid displacements $\boldsymbol{u}_f$ and tractions $\boldsymbol{\lambda}_f$ on the interface is defined as

$$\boldsymbol{\lambda}_f = F(\boldsymbol{u}_f) \quad (1)$$

This is also called Steklov-Poincare operator. It represents the process of mesh deformation and computation of flow field. Similarly, a Steklove-Poincare operator relating structure displacements $\boldsymbol{u}_s$ to tractions $\boldsymbol{\lambda}_s$ on the interface for the structural domain is defined by

$$\boldsymbol{\lambda}_s = S(\boldsymbol{u}_s) \quad (2)$$

The inverse operator of S is defined to map the interface tractions to displacements for the structure

$$\boldsymbol{u}_s = S^{-1}(\boldsymbol{\lambda}_s) \quad (3)$$

The interface matching is subject to two mechanics principles:

$$\begin{aligned} \boldsymbol{u}_{\mathrm{f}} = \boldsymbol{u}_{\mathrm{s}} = \boldsymbol{u}_{\Gamma}, \text{ on } \Gamma \\ \boldsymbol{\lambda}_{\mathrm{f}} + \boldsymbol{\lambda}_{\mathrm{s}} = 0, \text{ on } \Gamma \end{aligned} \quad (4)$$

The fluid tractions is computed by

$$\boldsymbol{\lambda}_f = -P \cdot \boldsymbol{n}_f + \tau \cdot \boldsymbol{n}_f \quad (5)$$

where P is the fluid pressure, τ is the viscid stress and $\boldsymbol{n}_f$ is normal vector on the fluid interface. Put Eq. (1) and Eq. (2) into (4), we can get

$$F(\boldsymbol{u}_f) + S(\boldsymbol{u}_s) = 0 \;\; , on \;\; \Gamma \quad (6)$$

we rewrite Eq. (6) as the form of Steklove-Poincare equation

$$F(\boldsymbol{u}_\Gamma) + S(\boldsymbol{u}_\Gamma) = 0 \quad (7)$$

Apply the inverse operator S-1 to this equation

$$\boldsymbol{u}_\Gamma = S^{-1}\left(-F(\boldsymbol{u}_\Gamma)\right) \quad (8)$$

Define an operator $g = S^{-1} \circ (-F)$, and Eq. 8 is transformed into the fixed-point formulation

$$g(\boldsymbol{u}_\Gamma) = \boldsymbol{u}_\Gamma \quad (9)$$

Thus the FSI iterations is expressed as the course of finding the solution u_Γ of the nonlinear Eq. (9) defined on Γ.

Time coupling

Once the partition approach is employed, there are two possible strategies for time coupling. The loose coupling or Conventional Serial Staggered method (CSS) [5] is widely used in FSI problems, For each time step, the Dirichlet-to-Neumann operator F and Neumann-to-Dirichlet operator S^{-1} are performed only once.

A prediction [12] of structure displacements is adopted at the beginning of the computation to improve the stability and accuracy. This method is called Generalized Serial Staggered (GSS). The predictor is defined by

$$P(\boldsymbol{u}_\Gamma^n) = \boldsymbol{u}_\Gamma^n + \alpha_0 \Delta t \dot{\boldsymbol{u}}_\Gamma^n + \alpha_1 \Delta t \left(\dot{\boldsymbol{u}}_\Gamma^n - \dot{\boldsymbol{u}}_\Gamma^{n-1}\right) \quad (10)$$

whereThe prediction is first-order time-accurate if α_0 = 1, and second-order time-accurate if α_0 = 1 and α_1 =1/2.

And the stability of these schemes highly depends on the density ratio ρ_s / ρ_f and the compressibility of the flow. To avoid this disadvantage, the strong coupling called fixed point method is suggested. The procedure of the fixed-point method is described in Algorithm 1, here k is the sub-iteration times.

Algorithm 1 Fixed-point method with relaxation

Data: start and end time (t0,t,max), time step Δt and initial interface displacement $\boldsymbol{u}_\Gamma^0$

while $t < t_{\max}$ do

Predict displacements: $\boldsymbol{u}_{\Gamma,p}^{n+1} = P(\boldsymbol{u}_\Gamma^n)$

k=0

while $k < k_{\max}$ do

FSI iteration: $\bar{\boldsymbol{u}}_{\Gamma,k}^{n+1} = g(\boldsymbol{u}_{\Gamma,k}^{n+1})$

Compute interface residual: $r_k^{n+1} = \bar{\boldsymbol{u}}_{\Gamma,k}^{n+1} - \boldsymbol{u}_{\Gamma,k}^{n+1}$

Update the interface displacements $\boldsymbol{u}_{\Gamma,k+1}^{n+1} = \boldsymbol{u}_{\Gamma,k}^{n+1} + \omega_k r_k^{n+1}$

if $\|r_k^{n+1}\| < TOL$ then

Break

else

k=k+1

end

end

$t = t + \Delta t, n = n + 1$

end

GEOMETRICALLY EXACT BEAM THEORY

To analyze the geometrically nonlinear dynamic motion of slender flexible structures, a suite of code based on geometrically exact beam theory has been developed by the authors [13, 14].

The configuration of a beam is described by defining a family of cross-sections the centroids of which are connected by a curve which is referred as the centerline of the beam. The space curve $\boldsymbol{\varphi}(\boldsymbol{S})$ is used to describe the centerline. The cross-sectional plane is defined by a triad of orthonormal basis $\{d_i\}_{i=1}^3$ which are attached to the cross-section and referred to as moving frame. The normal vector $\boldsymbol{d}_3$, as illustrated in Fig. 1.

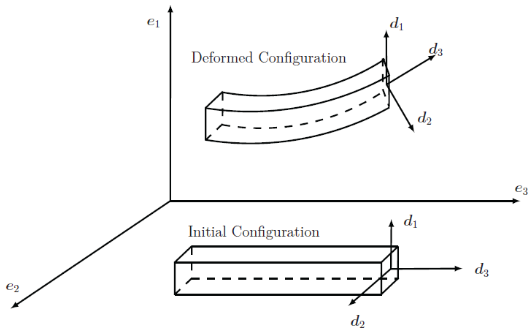


FIGURE 1: Initial and deformed configurations of the beam

Governing Equations

Let n(S) and m(s) denote the resultant force and resultant moment respectively, acting on the cross-section at S. Let $\bar{n}\left(S\right)$ and $\bar{m}\left(S\right)$ denote the prescribed body force and moment per unit of the reference length. Let p(S) and $\pi\left(S\right)$ be linear momentum and angular momentum. The well-known local forms of balance equations are given by

$$\begin{aligned} \dot{\boldsymbol{p}} &= n' + \bar{n} \\ \dot{\pi} &= m' + \varphi' \times n + \bar{m} \end{aligned} \quad (11)$$

The dot stands for the derivative of the time and the construction of the weak formulation of equilibrium equations is obtained by taking the dot product of Eq. (11) with an arbitrary admissible variation $\left(\eta,\nu\right)$, integrating over the domain [0, L] and using integration by part theorem. The result is

$$G\left(\varphi,\Lambda;\eta,\nu\right) = G_{int} + G_{dyn} - G_{ext} = 0 \quad (12)$$

Where G_{int} is elastic deform effect, $G_{\rm dyn}$ is the virtual work produced by the inertia of the beam and $G_{\rm ext}$ is the virtual work of the applied load and boundary stress resultants. For the linearization of the weak form, see the paper of Simo [11].

COMPUTATIONAL RESULTS

In this section, the open source CFD program Stanford University Unstructured (SU2) is used as fluid solver. In FSI problems, the fluid structure interfaces are moving boundaries, making it necessary to take mesh displacements into account. In SU2, the Navier-Stokes equations have been in an Arbitrary Lagrangian-Euler framework as follows:

$$\frac{\partial \mathrm{U}}{\partial \mathrm{t}} + \nabla \cdot \mathrm{F^c}\left(\mathrm{U}\right) - \nabla \cdot \mathrm{F^v}\left(\mathrm{U}\right) = 0 \quad (13)$$

where U is a conservative variable, $\mathrm{F^c}$ is convective fluxes and $\mathrm{F^v}$ is viscous fluxes.

To verify the application of geometrically exact beam in the FSI simulation, a 2D flexible cantilever mounted at the downstream face of a 2D square cylinder is studied as shown in Fig.2. The geometry parameters and physical properties are listed in Table 1.

The problem is computed in 2D flow of which Reynolds number with respect to D is Re = 332. The free stream Mach number was 0.2. For the time integration of the structural computation, the improved Newmark method [13] was adopted.

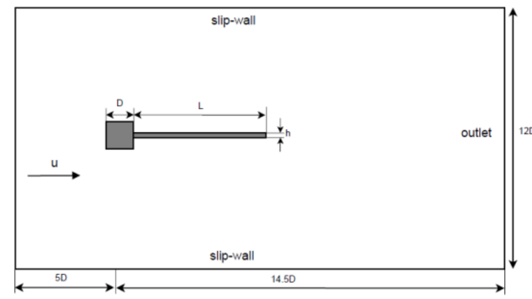


FIGURE 2: Flexible cantilever behind a square cylinder

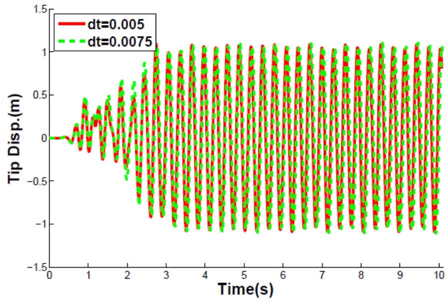
Table 1: Geometry and physical attributes of the flexible cantilever case

Geometry	Fluid	Structure
D=1m	$u_\infty = 51.3 m/s$	$E = 2.5\times10^6 Pa$
L=4m	$\rho_f = 1.18\times10^{-3} kg/m^3$	$\rho_f = 1.18\times10^{-3} kg/m^3$
h=0.06m	$\mu_f = 1.82\times10^{-4} kg/m\cdot s$	$\nu = 0.35$
h/L=0.015	Re=332	f_1=3.03Hz

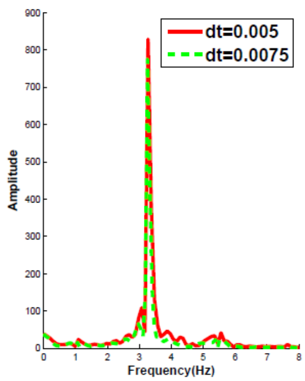
This problem was firstly proposed by Wall and Ramm [15] to verify their FSI coupled method. Two parameters that are usually used as the criterion are vibration frequency and amplitude of the cantilever tip. According to the previous studies, the amplitude ranges from D to 1.35D and the frequency is between 3.0Hz and 3.2Hz.

The coupling problem was computed with different time steps by the fixed-point method with dynamic Aitken's relaxation. The second-order predictor was adopted to estimate the interface displacements before each FSI iteration. In the simulation, the relaxation value increased rapidly and varied between 0.8 and 1.0. For most of FSI iterations, 5 or 6 steps were required to converge.

The time histories of the vertical displacement of cantilever tip and Fourier transform analysis are shown in Fig.3. For both time steps Δt_1 = 0.005s and Δt_2 = 0.0075s, the FSI iterations were enough to maintain the stability of the simulation. At the beginning of the simulation, the flow field was in a symmetric state and the cantilever was at rest. Due to the influence of the square cylinder, the flow field became unstable and the cantilever began to move, further affecting the flow field. After 3 seconds, the whole system reached to a stable state of oscillation. The vibration frequency is 3.2Hz, close to the first natural frequency f_1 = 3.*0*3Hz. Different computational results of previous studies are listed in Table 2. The result of this paper is in good agreement with those of other literatures.



(a) vertical displacements



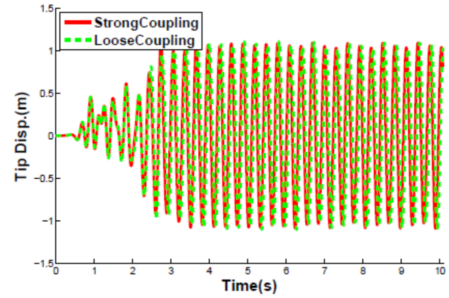
(b) FFT plots

FIGURE:3 TIME HISTORIES OF THE VERTICAL DISPLACEMENTS AT THE TIP OF THE CANTILEVER AND FFT PLOTS

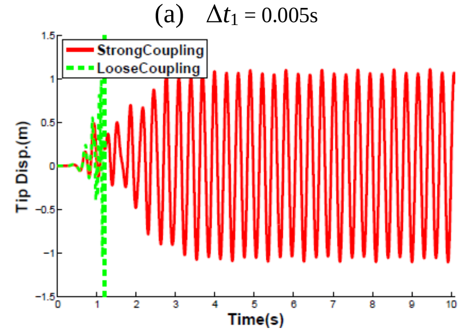
Table 2: COMPARISON OF VIBRATION FREQUENCY AND MAXIMUM TIP DISPLACEMENTS

Author	Frequency(hz)	Amplitude(m)*
Wall et al.	3.08	1.31
Sanchez et al.	3.05-3.15	1.15-1.21
Dettmer et al.	2.96-3.31	1.1-1.4
Habchi et al.	3.25	1.02
this paper	3.2	1.05

Time histories of tip displacements using different coupling methods are shown in Fig.4. For time step Δt_1 = 0.005s, there is small difference between the result computed by loose coupling without predictor and that computed by strong coupling with a second-order predictor. However, the maximum error of interface displacements computed by loose coupling method reached the magnitude of $O\left(10^{-1}\right)$, violating the kinematic continuity condition Eq.4. For time step Δt_2 = 0.0075s, the simulation performed by loose coupling without predictor goes to divergence after the tip displacements become relatively large. On the contrary, the simulation performed by strong coupling with a second-order predictor maintains good stability. Thus loose coupling method can only solve the coupled problem with small enough time step, while strong coupling method is capable of solving the problem with a larger time step.



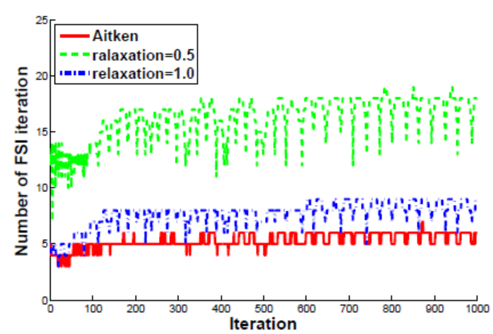
(a) Δt_1 = 0.005s



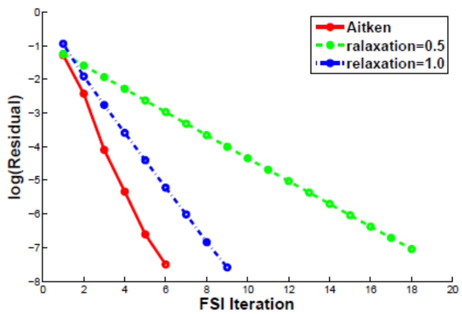
(b) Δt2 = 0.0075s

FIGURE:4 TIME HISTORIES OF TIP DISPLACEMENT USING DIFFERENT COUPLING METHOD

Finally different relaxation methods are investigated. The number of each FSI iterations for dynamic Aitken's relaxation, fixed relaxation $(\omega=0.5)$ and no relaxation $(\omega=1.0)$ are shown in Fig.5. When the cantilever begins to vibrate, the methods with fixed relaxation value $(\omega=0.5 \; and \; \omega=1.0)$ usually requires more steps to converge than the method with dynamic Aitken's relaxation. Therefore, Aitken's relaxation is of great efficiency to speed up the FSI computation.



(a) Number of FSI iterations



(b) Residual histories at a certain step

FIGURE:5 Comparison of dynamic Aitken's relaxation, fixed relaxation and none relaxation

CONCLUSIONS

In this paper, the computation of FSI problem is viewed as finding the solution of the interface Steklov-Poincare equation, which is solved by the fixed-point method with Aitken's dynamic relaxation. The coupling methods commonly used in computational aeroelasticity in previous researches are compared with the fixed-point method, which is not difficult to implement. Using SU2 as the fluid solver, we verify a typical FSI problems in transonic and low speed fields. The problem of vortex induced vibration of a cantilever is computed using the geometrically exact beam solver and proves the feasibility of Simo's beam theory in the analysis of FSI problems. The results show that the fixed-point method is of good performance in accelerating the convergence and also maintains stability for a relatively large computational time step in the FSI iterations.

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