DRAFT: CYLINDER EFFECT ON THE FLAPPING OF AN INVERTED PLATE IN A UNIFORM FLOW

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ABSTRACT

The flapping dynamics of an inverted plate in a uniform flow has been widely used to study the energy harvesting system stability and efficiency. Different motion modes have been observed in previous numerical studies when the plate vibrates freely without any obstacles in proximity. This paper uses immersed boundary method (IB) to numerically study the cylinder effect on the flapping motion of an inverted plate, investigate the effect of the Reynolds number, the flexibility and distance, and relevant symmetric/asymmetric flow patterns, research the influence of vortex generating and shedding to the force change, explore the key factors and reveal the mechanism of different modes. Three vibration modes: straight, flapping and deflected will be affected differently by the distance to the surface. The maximum mean strain energy is found at a certain distance. Such research could theoretically support the development of energy harvesting system.

NOMENCLATURE

$\beta$  Bending stiffness
$\mu$  Mass ratio
$Re$  Reynolds number
$D$  Plate-cylinder distance

INTRODUCTION

Scientists have been studying the vibration of a slender body in uniform flow for the potential application of energy harvesting device [1]. Fluid and vortex induced vibration of the flexible body can absorb and transfer the energy to electricity by means of piezoelectric materials attached to the structure surface [2]. Such is a typical fluid-structure interaction problem, so it is important to understand how the plate vibrate to deform largely and harvest the most energy from the flow. Experimental and numerical researches have been conducted to investigate the fundamental mechanism of fluid-structure interaction in this problem [3, 4]. Researchers place the plate in the free flow, fix its trailing edge and free the leading edge, which makes the plate flap inversely and change the flow pattern. Different flexibility and density as well as the
vortex shedding have been analyzed to investigate the vibration characteristics [5, 6]. Besides vibrating in the free flow, the plate can also vibrate induced by the vortex shedding from other objects such as cylinder. Some researches have been conducted to study the cylinder effect on the plate flapping inversely in the flow to generate electricity [7]. However, it still lacks further investigation and the characteristic factors. This paper intends to use numerical method to simulate the plate vibrates behind a cylinder and conduct parametric study to explore how the key factors affect the deformation.

**PROBLEM DESCRIPTION AND NUMERICAL METHOD**

In this paper, a two-dimensional plate inversely vibrating behind a cylinder is numerically studied. The plate with length $L$ and negligible thickness freely flaps at its leading edge and is clamped at the trailing edge as a cantilever beam as shown in Fig. 1. The uniform flow with velocity $U$ passes the cylinder with diameter $L$ to generate wake and induce the plate to vibrate passively. The distance between the leading edge and closest cylinder surface point is marked as $D$. The plate is assumed to be elastic and inextensible, and its dynamics is governed by the Eqn. 1 [8]

$$\rho_s h \frac{d^2 x}{dt^2} = \frac{\partial}{\partial l} \left( \tau t + q n \right) + f$$

where $\rho_s$ and $h$ are the plate density and thickness, respectively, $t$ denotes the unit tangent vector in the direction of increasing arc length $l$, $n$ is the unit normal vector and $f$ is the force distributed load difference on two sides of the plate. The in-plane tension $\tau$, is assumed to be proportional to the tangential strain as

$$\tau = E \left( \left| \frac{\partial x}{\partial l_0} \right| - 1 \right)$$

The plate is set as inextensible along the arc length limited by the condition

$$\frac{d(\partial x/\partial l)}{dt} = 0$$

The transverse stress $q$ is linearly related to the bending moment $M$ as

$$q = \frac{\partial M}{\partial l} = \frac{\partial E_B \kappa}{\partial l}$$

where $E_B$ is the bending modulus and $\kappa$ is the curvature. The boundary condition at the leading edge $l = 0$ is specified as zero bending moment and transverse stress, which means $\kappa = 0$ and $\partial \kappa / \partial l = 0$, such that

$$\frac{\partial^2 x}{\partial l^2} = 0, \quad \frac{\partial^3 x}{\partial l^3} = 0$$

At trailing edge $l = L$, the translational and rotational boundary conditions are

$$x = x_0, \quad \frac{\partial x}{\partial l} = \left( \cos \alpha, -\sin \alpha \right)$$

where $\alpha$ stands for the inclination between the unperturbed plate and the flow direction.

The flow is governed by the viscous incompressible Navier–Stokes equation and the continuity equation,

$$\frac{\partial v_i}{\partial t} + \frac{\partial v_j v_i}{\partial x_j} = - \frac{1}{\rho_f \partial x_i} \frac{\partial p}{\partial x_i} + v_f \frac{\partial^2 v_i}{\partial x^2_i},$$

$$\frac{\partial v_i}{\partial x_i} = 0,$$

where $v_i$ is the velocity, $\rho_f$ and $v_f$ are the fluid density and viscosity, and $p$ is the pressure. No-slip and no-penetration conditions are specified at the flow–solid boundary.
The equations governing the system, e.g. Eqn. (7), are solved numerically in an implicitly coupled manner using an in-house solver. Specifically, the incompressible flow is solved using a sharp-interface immersed-boundary method [9, 10] with a special treatment to suppress the pressure oscillations associated with the moving boundaries [11]. In this method, a single-block Cartesian grid is used to discretize the Navier–Stokes equation on a rectangular domain, and the ghost nodes and hybrid nodes are defined near the fluid–solid interface to facilitate the boundary treatment at the interface (Fig. 2). The infinitely thin plate is augmented with an artificial thickness that is about three times of spacing of the Cartesian grid and is automatically reduced as the grid is refined. The plate section is discretized by a set of Lagrangian points initially distributed uniformly along the plate.

To parametrize the system, we define the non-dimensional group of parameters including the normalized distance, mass ratio, plate flexibility, Reynolds number, which are given by

\[ D^* = \frac{D}{L}, \quad \mu = \frac{\rho_s h}{\rho_f L}, \quad \beta = \frac{E_B}{\rho_f U^2 L^3}, \quad Re = \frac{UD}{\nu_f}, \tag{8} \]

In the present research, we choose the Reynolds number \( Re = 300 \) mass ratio \( \mu = 5 \) and rotational angle \( \alpha = 0 \) to focus on the flexibility and distance effect on the plate flapping motion and how the energy is transferred. The

**FIGURE 2**: A 2d illustration of the sharp-interface immersed-boundary method.

**FIGURE 3**: Peak-to-peak amplitude with respect to \( D^* \), \( \beta = (A) 0.05; (B) 0.15; (C) 0.6. \)
The flexibility factor $\beta$ is selected as 0.05, 0.15 and 0.6 corresponding to straight, flapping and deflected modes respectively, if no cylinder involved. The distance $D^*$ ranges from 0.5 to 6 to The computational domain is set as $20L \times 10L$, consisting of a nonuniform Cartesian grid of $512 \times 296$ points. The core domain $[-2L, 2L] \times [-2L, 8L]$ is uniformly distributed in vertical and horizontal directions respectively with grid spacing $\Delta x = \Delta y = 0.025L$. The flexible plate is discretized with evenly distributed 100 nodes. Time step is $\Delta t = 0.0125T$, where $T = L/U$ is the characteristic time. The flow and structure solvers are validated in our previous papers as well as the grid-independent study [12].

**RESULTS AND DISCUSSION**

**Vibration Dynamics**

When the plate flaps symmetrically and periodically, the energy is best converted. Such phenomenon is related to the flexibility, inertia and vortex shedding, among which inertia plays a minor role. The peak-to-peak amplitude is shown in Fig. 3 with vibration amplitude of single plate as reference. $\beta = 0.6, 0.15$ and 0.05 corresponds to three different dynamic modes: straight, flapping and deflected respectively in Fig. 4(A)(B)(C)([5]). For the straight and deflected modes ($\beta = 0.6$ and 0.05), the plate maintains minimum vibration with small amplitude, which is not suitable for the energy conversion from fluid. Fig. 3(A)(C) show that the vibration amplitude is evidently enhanced when the plate inversely vibrates behind the cylinder. The curves shows that as the distance $D^*$ is greater than 1, the vibration amplitude increase is more obvious. For $\beta = 0.05$, the plate still deforms in the the quasi-steady deflected mode. For the low flexibility of $\beta = 0.6$, the plate begins to flap symmetrically and periodically by the wake effect, which enhances the amplitude greatly. Such change could make the energy harvesting efficient for stiffer structure. However, when $D^*$ is less than 1, the vortex can not shed completely from the cylinder and enclose the plate inside the wake development zone, which constrains the vibration. For $\beta = 0.15$, Fig. 3(B) shows that the vibration amplitude actually does not exceed the that of single plate under the effect of cylinder. When the plate stays close to the cylinder, the vibration is heavily constraint. As the $D^*$ reaches more than 3, the vibration amplitude gradually approaches the value of a single plate.

**Vortical Structures**

The results are shown in Fig 5, which display the vortex shedding and the plate vibration for different flexibility and distance. When the plate is too close as in Fig 5(A)(C)(E), the cylinder and plate combine as a entire entity and vortex is formed and shedding from the
trailing edge, which on the contrary limits the vibration for all three $\beta$. As the distance increases, the vibration is greatly induced and enhanced by the shedding vortex compared to the single plate. The vortex from cylinder interacts and merges with the plate to transfer the energy.

CONCLUSION
The paper numerically investigates the plate inversely vibrating behind a cylinder. It shows that the cylinder can affect the plate vibration greatly and totally change its vibration mode. For $\beta = 0.6$, the effect increases the vibration mostly when $D^* >= 3$ as well as the vibration mode changes from deflected to flapping state. However, for $\beta = 0.15$ the vibration is somehow decreased by the shedding vortex. Further investigation is needed to explore the mechanism of unsteadiness and bifurcation. Such study can be further compared with three dimensional research and experiment test to get deeper understanding of fluid-induced vibration and help invent the novel energy harvesting system.

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REFERENCES


