DRAFT: LOAD TRANSFER IN AEROELASTICITY BASED ON RADIAL BASIS FUNCTIONS

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ABSTRACT
Load transfer is of importance in simulation of aeroelasticity by coupling fluid forces with the structure deformations. RBFs with different forms and parameters are tested in load transfer and their results are compared in this paper. Results of comparisons show that different RBF’s with will lead different results. For globally supported RBF’s, more accurate loads on structure can be gained by MQB and TPS functions. And for compactly supported RBF’s, wisely chosen support radius should be used to gain proper results.

NOMENCLATURE
\(f_f\) aerodynamic loads  
\(f_s\) interpolated loads on structure  
\(\delta u_s\) displacements of structure  
\(\delta u_f\) displacements of fluid boundary grid  
\(H\) loads transferring matrix  
\(d\) dimensions of problem  
\(N_f\) number of fluid boundary points  
\(N_s\) number of structure points  
\(u_f\) displacements of structure points  
\(u_f\) displacements of fluid boundary points

INTRODUCTION
With subsonic civil transports operating in the transonic regime, it is important to determine the coupling between the aerodynamic loads and elastic deformations or flutter of aircraft structures or wings. Since aeroelasticity contributes significantly to the design of an aircraft, there is a strong need in the aerospace industry to predict these fluid-structure interactions computationally.

To analyze static aeroelastic problems under transonic condition, fluid structure interaction should be applied with computational fluid dynamics (CFD) analysis tools coupling with finite element (FE) analysis tools. The CFD and FE code have been well developed by which the nonlinear behavior of aerodynamic and structure can be predicted. Since the source codes are not always available, the coupling process will run CFD and FE codes by turn and transfer loads and deformations between aerodynamic grid and finite element mesh [1, 2]. The loads transfer is of importance since the transferred loads on structure should be precisely reflect the loads on aerodynamic grid.

The loads transfer has been implemented with two different strategies. The first strategy focus on the relative position of aerodynamic grid and structure mesh so that geometry relation information needs to be finding out in transfer [3, 4]. This class of methods is very accurate in some specific problem; however it is difficult to be used...
automatically or to be applied on non-match grid. The second strategy based on energy conservative law and the topological relation between aerodynamic grid and structure mesh is unrelated so that it is easy to transfer loads automatically [5,6]. Nonetheless the loads transfer might be difficult to control, which means the energy is conservative globally but the local distribution could be non-match.

The radial basis functions (RBF’s) interpolation is a well established tool for conservative interpolation and variant basis functions can be chosen in fluid-structure-interaction. The object of this research is to find a best way to do the RBF interpolation by comparing variant basis functions and parameters for static aeroelastic problem of a civil transporter wing in transonic regime.

ENERGY CONSERVATION
Since the field of aerodynamic and structure are discretized by different mesh due to the different physical characteristics of fluid and structure, their boundaries do not match each other. In another word, they do not share same nodes on boundaries so that the discretized field of structure with lesser grid points could differ significantly from the smooth aerodynamic surface mesh with much more points. The structure model can be built as finite element model in Lagrange formulation meanwhile the fluid can be built as volume and surface grid in Euler formulation. The interpolation should convert the aerodynamic loads \( f_f \) to the structure forces \( f_s \), and then transfer the displacements of structure \( \delta u_s \) caused by forces \( f_s \) to the displacements of fluid boundary grid \( \delta u_f \).

Since the works performed by aerodynamic forces and structure stresses should keep equivalent during coupling meanwhile the aerodynamic forces counterbalance with structure stresses, the energy conservation law show as:

\[
\delta w = \delta u_s^T \cdot f_f = \delta u_s^T \cdot f_s = \delta u_f^T \cdot f_s
\]  

A couple matrices \( H \) could be introduced to connect the displacement of fluid and structure with approximated linear relation:

\[
\delta u_f = H \cdot \delta u_s
\]  

Then the linear relation between aerodynamic forces and structure forces can show as:

\[
f_s = H^T \cdot f_f
\]  

If the coupling matrix \( H \) can be constructed, the matrix and its transpose can be used for transferring displacements from structure to fluid and transferring aerodynamic forces to structure forces [3].

RADIAL BASIS FUNCTION INTERPOLATION
RBF’s is a widely used multivariable interpolation tool for scatter data. Besides the fluid structure interaction, the RBF also play an important role in engineering area, such as image processing, Computer-Aided Design (CAD), Computer-Aided Engineering (CAE) [7, 8, 9]. The application of RBF in fluid-structure-interaction will be discussed in this part.

Generally, the multivariable interpolation can be described as following. A sample of multivariable scatter data is assumed to be given as a set of points \( X = \{x_1, \cdots x_N\} \subseteq \mathbb{R}^d \) and a set of values on these points given respectively as \( u_1, \cdots u_N \). To construct a function that can generate values \( u_{1}, \cdots u_{N} \) only from point evaluation, an interpolation function \( s_{u,X}(x) \) can be expressed as:

\[
s_{u,X}(x) = \sum_{i=1}^{N} a_i \phi(||x - x_i||) + p(x) \tag{4}
\]

The interpolation basis function \( \phi(||x||) \) is RBF only in respect of Euclidean distance and \( p(x) \) is polynomial with number \( d \) of variables. These two parts to be determined should satisfy following conditions [10]:

1. The value of interpolation function at point \( x_j \) equals to known value \( u_j \):

\[
s_{u,X}(x_j) = u_j, \ 1 \leq j \leq N \tag{5}
\]

2. For any polynomial \( q(x) \) with lower or equal order than that of \( p(x) \):

\[
\sum_{j=1}^{N} a_j q(x_j) = 0 \tag{6}
\]

The interpolation will be unique if the basis function is conditionally positive definite. The linear polynomial \( p(x) = a_0 + a_1 x + a_2 y + a_3 \) could be applied if the basis function is conditionally definite positive with order lower than two. For fluid structure integration, the linear polynomial is adaptable and Euclidean space is limited as 3-dimension.

For the fluid-structure-interaction problem, the forces which are pressures on \( N_f \) fluid grid points \( X_f = (x_j^f, y_j^f, z_j^f) \), \( 1 \leq j \leq N_f \) need to be interpolated as \( N_s \) forces on structure points \( X_s^f = (x_j^f, y_j^f, z_j^f) \), \( 1 \leq j \leq N_s \). Also, the displacements of fluid points \( u_j^f \) need to be recovered from structure points’ displacements \( u_j^s \). The structure points could be setup as the set of \( X \) in the multivariable interpolation and displacement as the function value to be evaluated. The coefficients of the constructed interpolation function could be solved by linear equations as following:

\[
\begin{bmatrix}
0 \\
u_j^s
\end{bmatrix} =
\begin{bmatrix}
0 \\
p M^T \omega
\end{bmatrix}
\]

\( u_j^s \) is the displacement of known structure points. \( \omega \) is a vector consisting of coefficients \((\omega_0, \omega_1, \omega_2, \omega_3)^T\) in \( p(x) \). \( \alpha \)
is the vector of coefficients of interpolating basis functions, expressed as \((a_1, \ldots, a_{N_f})\). \(M\) is a sub matrix with size of \(N_x \times N_y\) and the elements of it is radial basis functions 
\[
\phi_{ij} = \phi(\|x_i^j - x_j^j\|) \quad P \text{ is a } N_s \times 4 \text{ matrix with rows expressed as } (1, x_j, y_j, z_j).
\]

The coefficients \([\omega \alpha]^T\) of basis functions could be solved by linear Eq. (7) to determine the interpolating function \(s_{u,x}(x_j)\) and further more to gain the displacements of fluid points \(u^f_j = s_{u,x}(x_j^f)\).

The transferring matrix in Eq. (2) then could be expressed as \(H = C_{ss}^{-1}\). \(C_{ss}\) is a matrix consisting of coefficients of \(A_f\) and \(A_{fs}\) expressed as following matrix:

\[
A_{fs} = \begin{bmatrix}
1 & x_1^f & y_1^f & z_1^f & \phi_{1,1}^s & \cdots & \phi_{1,N_s}^s \\
1 & x_2^f & y_2^f & z_2^f & \phi_{2,1}^s & \cdots & \phi_{2,N_s}^s \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
1 & x_{N_f}^f & y_{N_f}^f & z_{N_f}^f & \phi_{N_f,1}^s & \cdots & \phi_{N_f,N_s}^s
\end{bmatrix}
\]

In which \(\phi_{ij}^s = \phi(\|x_i^f - x_j^f\|)\). \(C_{ss}^{-1}\) represents the determination of interpolating coefficients, and \(A_{fs}\) represents the basis functions of interpolation. The matrix \(H\) will be the loads transferring matrix.

**CHOICE OF RADIAL BASIS FUNCTIONS**

RBFs could be categorized as compactly [10, 11, 12] and globally supported functions.

Globally supported RBF’s will lead values not equal to zero in whole domain that cover the interpolating space. The common RBF’s are listed in Table 1. The MQB method uses a parameter \(a\) to control the shape of the basis function. Larger \(a\) leads a more flat shape while smaller \(a\) makes the function more sharp. Generally the value of \(a\) is chosen in range of \(10^{-5}\sim10^{-3}\) [13]. The effects caused by different value of \(a\) will be investigated in this paper.

**Table 1: Globally supported RBF’s**

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>(f(|x|))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thin plate spline (TPS)</td>
<td>(|x|^2 log(|x|^2))</td>
</tr>
<tr>
<td>2</td>
<td>Gaussian (Gauss)</td>
<td>(e^{-|x|})</td>
</tr>
<tr>
<td>3</td>
<td>Multiquadric biharmonics (MQB)</td>
<td>(\sqrt{a^2 + |x|^2})</td>
</tr>
<tr>
<td>4</td>
<td>Quadric biharmonics (QB)</td>
<td>(1 + |x|^2)</td>
</tr>
</tbody>
</table>

The compactly supported RBF’s characterized by

\[
\phi(\|x\|) = \begin{cases}
    f(\|x\|) & 0 \leq x \leq 1 \\
    0 & x > 1
\end{cases}
\]

The value of RBFs can be compulsorily set to non-zero inside a circle or sphere with radius of \(r\) and to zero outside, by defining \(\phi_r(x) = \phi(\|x\|^2/r^2)\). The region that a point affects can be controlled by setting \(r\) to variant values, which means the local characteristic will be easily controlled if the compactly supported RBF’s are used. Common compactly supported RBF’s are listed in Table 2 [14].

**Table 2: Compactly supported RBF’s**

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>(f(|x|/|x|))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CPC0</td>
<td>((1 - |x|)^2)</td>
</tr>
<tr>
<td>2</td>
<td>CPC2</td>
<td>((1 - |x|)^2(4|x| + 1))</td>
</tr>
<tr>
<td>3</td>
<td>CPC4</td>
<td>((1 - |x|)^6\left(\frac{35}{3|x|} + 6|x| + 1\right))</td>
</tr>
<tr>
<td>4</td>
<td>CPC6</td>
<td>((1 - |x|)^8\left(32|x|^3 + 25|x|^2 + 8|x| + 1\right))</td>
</tr>
</tbody>
</table>

Theoretically the compact support radius should be set to different value on different interpolating points, but it could lead to singular matrix. So uniform radius will be chosen for all points in one case [15]. The radius also need to be chosen carefully to cover enough points locally while make no effect on points that far away. The influence of radius will be investigated in this paper.

**TEST CASES AND RESULTS**

A typical transport wing from CHNT-1 (China Transport-One is a large passenger aircraft model designed by China Aerodynamics Research and Development Center) is adopted as aerodynamic model. The fluid field is discretized by structured grid and wall boundary condition considers the viscosity effect. The aerodynamic analysis is performed at Mach number of 0.78 and height of 11000 ft using CFX solver based on Reynolds-Averaged Navier-Stokes (RANS) equations. The calculated pressure is shown in Fig. 1 (A).

The structure model simply consists of beams and the finite element model is established all by CBEAM elements, shown in Fig. 1 (B). It can be described as a stiffness axis and beams attached on it. The stiffness axis is clamped at root and has a constant section.

For a wing and its structure, the bend and twist deformations, which produced by bending moment and torque relative to stiffness axis, will significantly affect the aerodynamic performance. So the consistency of torque offered by fluid pressure and torque interpolated from pressure, which will exert on structure, is considered as important criterion to evaluate the interpolating capabilities of different RBF’s. All RBF’s in Table 1 and CPC0 and CPC2 in Table 2 are chosen to be tested.

Three values are chosen for the shape control parameter of MQB, \(a = 1e - 3\), \(a = 1e - 4\), \(a = 1e - 5\). The radius of compactly supported RBF’s also varies in three values. The first two are maximum and minimum distance between any two
mesh points, say R=18.0 and R=2.0. The third is a value between max and min, say R=9.0. The field for interpolation will also be cut into pieces and interpolating in split area, by which the interpolating region will be narrowed thus the inconsistence could be limited.

The interpolated torque about to be exerted on structure and original torque produced by fluid pressure are shown in Fig. 2. The original torque distribution can be integrated from the fluid mesh and keep relative high precision because of dense CFD grid. The results calculated by RBF’s with different parameters are shown in sub Figure 2 (A-C). Figure 2 (D-F) show the results using same RBF’s and parameters on split region. Results of globally supported RBF’s are only shown in Fig. 2 (A, D) and comparisons with different shape control parameters for MQB method are shown in Fig. 2 (B, E). Results of varying support radius for compactly supported RBF’s are shown in Fig. 2 (C, F).

The results show that TPS and MQB function with varying parameters have capabilities to interpolate torque distribution that well match with original torque. QB and Gauss show similar trend to the original torque but not approximately match. The RBF’s with compactly support do not show well results that expected. The distribution of torque interpolated by compactly supported RBF’s looks like that scatter data spread around the original distribution. The results interpolated on split regions show better agreement. If we differentiate them carefully on split regions, the compactly supported RBF’s will get better results when radius set to 9 between max and min. For MQB function, the shape control parameters in suggested range do not apparently affect the results.

The interpolated bending moment distributions are shown in Fig. 3 (A), comparing with those of original bending moments. All the globally supported RBF’s show good accuracy of interpolation, while the compactly supported RBF’s act better when medium value is set for supporting radius. Especially, the CPC0 function shows bad approximation if the supporting radius set to a minimum value. The results on split regions are shown in Fig. 3 (B) and are all close to the original distributions, which indicate that better interpolation could be gain from split regions.

**FIGURE 1:** Aerodynamic model and structure model of CHNT-1.

![Aerodynamic model and structure model of CHNT-1](image1.png)

![Graph showing interpolated torque](image2.png)

![Graph showing torque distribution](image3.png)

![Graph showing torque comparison](image4.png)
FIGURE 2: Comparison of torque distribution. (A-C) for common RBF's, (D-F) for RBF's interpolating on split regions.

FIGURE 3: Comparisons of bending moment distribution. (A) for common RBF's, (B) for RBF's interpolating on split regions.

CONCLUSIONS

In this paper, a loads transfer method based on RBF's, which can be applied in coupling of fluid and structure, is developed. The results calculated from several common types of RBF's are thoroughly compared. The comparisons indicated that RBF's are very accurate tools to transfer loads from fluid grid to structure mesh by interpolation if the parameters are chosen properly. The TPS and MQB act best in interpolating, and other RBF's could be limited in small split regions to gain better results. Supporting radius needs to be considered carefully if compactly supported RBF's are used. The medium value between maximum and minimum distance will gain better results than the two borders of the range, which will lead totally obviously inconsistent between interpolated and original distributions of torques. The results could be practically valued in engineering and provide reference for coupling between fluid and structure, which is widely applied in fluid structure interaction simulation.

REFERENCES


