VIBRATION RESPONSES OF TWO TANDEM CYLINDERS OF DIFFERENT NATURAL FREQUENCIES

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ABSTRACT
The paper presents an experimental investigation on characteristics of flow-induced vibrations of two tandem cylinders of different natural frequencies. Both cylinders are allowed to vibrate in the cross-flow direction only. Six different natural frequency ratios \( f_n^* = \frac{f_{n,u}}{f_{n,d}} \) are considered, where \( f_{n,u} \) and \( f_{n,d} \) are the natural frequencies of the upstream and downstream cylinders, respectively. The spacing ratio \( L/D \) (where \( L \) is the spacing between the two cylinder centers and \( D \) is the diameter) is considered as 1.5 and 2.0. Simultaneous measurements of vibration and frequency responses and vortex shedding frequencies are conducted using laser vibrometers and hotwires, respectively. The results indicate that in the galloping vibration regime there is a critical reduced velocity at which the amplitude of the downstream cylinder drastically jumps and that of the upstream cylinder may drop, depending on \( f_n^* \). The jump/drop is connected to a lock-in of the vortex shedding with the fifth harmonics of \( f_{n,d} \). The different natural frequencies of the cylinders may suppress both vortex-excited and galloping vibrations of the cylinders at different reduced velocity ranges. The vibration response and its connections with the frequency ratio are elucidated. How the different natural frequencies of the upstream and downstream cylinders affect the vortex shedding frequency in the wake is also illustrated.

INTRODUCTION
Combinations of multiple cylindrical structures appear widely in various engineering fields, such as risers, undersea pipelines, masts, groups of chimney stacks, transmission line bundles, chemical reaction towers, etc. Two tandem cylinders may be a representative model that can render us an insightful knowledge of the flow around multiple cylindrical structures.

Flow induced vibrations (FIV) of two elastic tandem cylinders have been extensively studied in the literature. When the two cylinders are free to vibrate in two-degree of freedoms, King and Johns [1] performed experiments in a water tunnel for cylinder center-to-center spacing ratio \( L/D = 1.25 – 7.0 \) at a mass-damping ratio \( m^*\zeta = 0.051 \), where \( D \) is the cylinder diameter, \( m^* \) is the cylinder mass ratio, and \( \zeta \) is the structural damping ratio. At \( L/D = 2.5 \), the upstream cylinder response shows a typical vortex excitation (VE) pattern. The maximum amplitude for the downstream cylinder occurs at a reduced velocity \( U_r (= \frac{U_\infty D}{f}) \) = 7.7 (where \( U_\infty \) is the freestream velocity and \( f \) is the natural frequency of the cylinder system), with the amplitude decreasing exponentially after the maximum. Huera-Huarte and Bearman [2] conducted experiments on FIV of two tandem cylinders for \( L/D = 2.0 – 4.0 \) at \( m^*\zeta = 0.043 \). The upstream cylinder experiences a larger vortex-excited vibration than the downstream one for \( L/D = 2.0 – 2.5 \) at \( U_r = 4 – 9 \) where vortex shedding frequency \( f_s \) is close to \( f_n \). At \( L/D = 3.0 – 4.0 \), the downstream cylinder exhibits galloping vibrations for \( U_r > 9 \). With \( L/D \) varying from 1.2 to 6.0, vibration responses of the cylinders are systematically measured by Sun et al. [3] for a larger range of \( U_r (= 3.8 – 47.8) \). Based on the characteristics and galloping vibration generation mechanism, the vibration responses are classified into four regimes. Regime I (\( L/D \leq 1.5 \)) is characterized by both cylinders experiencing galloping vibrations and the downstream cylinder vibration amplitude smaller than the upstream cylinder. At Regime II (\( 1.5 < L/D < 2.5 \)), the galloping vibration is larger for the upstream cylinder than the downstream cylinder at smaller \( U_r \), but the opposite prevails at larger \( U_r \). At Regime III (\( 2.5 \leq L/D \leq 3.0 \)), the downstream cylinder vibration amplitude is larger than the upstream cylinder. Regime IV (\( L/D > 3.0 \)) features small
vibration for the downstream cylinder and no vibration for the upstream cylinder.

The past investigations were mostly concerned with two identical tandem cylinders which have the same natural frequency. However, the adjacent cylindrical structures may always not have the same natural frequency but may be of different frequencies. To the best of the authors’ knowledge, there is not a single systematic study of the flow-induced vibration of two cylinders (staggered, side-by-side or tandem) of different natural frequencies. Thus, a number of issues might arise. For instance, what is the effect of different natural frequencies of two elastic tandem cylinders on their vibration responses? While vibrations of two cylinders of an identical natural frequency are dependent on each other, can two cylinders vibrate at two different natural frequencies, respectively, given the two cylinders interacting each other? How do the different vibration frequencies influence the downstream vortex shedding frequency? This work aims to experimentally investigate FIV responses of two tandem cylinders of different natural frequencies, where both cylinders are free to vibrate in the cross-flow direction. Two $L/D (= 1.5$ and $2.0)$ are chosen from regimes I and II, respectively, based on the vibration regimes in Sun et al. [3]. Vibration responses of the cylinders are systematically measured for a large range of the incoming flow velocity $U_{\infty} = 0.6 – 16 \text{ m/s}$, corresponding to $Re = 1.2 \times 10^3 – 3.2 \times 10^4$. The turbulent intensity for this range of velocity was less than 0.2%.

To change the natural frequency $f_n$ of the cylinder system, five different coil-spring systems of different spring constant $k$ ($= 0.48 \text{ N/mm}, 1.04 \text{ N/mm}, 1.24 \text{ N/mm}, 1.64 \text{ N/mm}, 2.80 \text{ N/mm}$) were adopted. The $\zeta$ and $f_n$ were 0.0013 and 9.4 Hz for $k = 0.48 \text{ N/mm}$, respectively, yielding $m^*\zeta = 0.59$. The $\zeta$ for different $k$ essentially nests in the range of 0.0013 – 0.0016; the range is small and the effect of changing $\zeta$ can be ignored. The different combinations of spring systems for the upstream and downstream cylinders can lead to different natural frequency ratios $f_n^* (= f_{n,u}/f_{n,d})$, where $f_{n,u}$ and $f_{n,d}$ are the natural frequencies of the upstream and downstream cylinders respectively. Six combinations corresponding to $f_n^* = 0.6, 0.8, 1.0, 1.2, 1.4$ and $1.6$, respectively, are chosen. One hotwire (HT) was used to measure the streamwise fluctuating velocity $u$. The measurements of $Y^*$, $Y$ and $u$ are simultaneous to have a mutual discussion.

FIGURE 1: (a) EXPERIMENTAL SETUP, (b) DEFINITION OF SYMBOLS, (c) THE CYLINDER SUPPORT SYSTEM.

EXPERIMENTAL SETUP

Experiments were performed in a low-speed, closed-circuit wind tunnel. The two cylinders were mounted in tandem. Figure 1 shows a schematic of the experimental setup. Both cylinders were with the diameter $D = 30 \text{ mm}$ and length $l = 540 \text{ mm}$. The mass ratio $m^*$ was 453. As shown in Fig. 1, each cylinder end consisted of two 1-mm-thick leaf springs made of brass and two coil springs made of steel, allowing the cylinder to vibrate in the cross-flow direction only. Note that the two cylinders were mounted independently. To avoid interference/complexities caused by the cylinder ends, end plates were fitted on both sides. Each end plate consisted of two rectangular slots of $12\text{ mm} \times 60\text{ mm}$ to ensure an enough clearance for the cylinder vibration. The flow-induced displacements $Y'$ and $Y$ of the upstream and downstream cylinders, respectively, were measured simultaneously by using two laser vibrometers. The cylinder displacement amplitudes $A_u$ and $A_d$ were obtained from the rms (root-mean-square) value of $Y'$ and $Y$, i.e., $A_u = Y'_\text{rms} \times 2$ and $A_d = Y_{\text{rms}} \times \sqrt{2}$. The freestream velocity $U_{\infty}$ was varied from 0.6 to 16 m/s, corresponding to $Re = 1.2 \times 10^3 – 3.2 \times 10^4$. The turbulent intensity for this range of velocity was less than 0.2%.
RESULTS AND DISCUSSIONS

Vibration and frequency response characteristics at $L/D = 1.5$

Figure 2 shows the dependence of $A_u^*$ ($= A_u/D$) and $A_d^*$ ($= A_d/D$) on $U_{ru}, U_{rd}$ and $f_n^*$. Since the upstream and downstream cylinders vibrate dominantly at their natural frequencies $f_{ru}$ and $f_{rd}$, respectively, two reduced velocities $U_{ru} (= U_{ru}/f_{ru}D)$ and $U_{rd} (= U_{rd}/f_{rd}D)$ in correspondence with the upstream and downstream cylinders, respectively, are presented on the horizontal axis. The $f_n^* = 1.0$ case, considered as base results, is discussed first (Fig. 2c). Both $A_u^*$ and $A_d^*$ begin to increase at $U_{rd} = 5.3$. The small vibration ($A_u^* \approx 0.05$) at $U_{rd} = 5.3 – 7.2$ is VE caused by a lock-in. The $A_u^*$ with $U_{rd}$ increasing escalates to 0.31 – 0.37 at $U_{rd} = 8.0 – 13.7$, which is connected to the galloping vibration (Sun et al. [3]). The observation suggests a combined VE and galloping vibrations. The $A_u^*$ decreasing to about 0.14 at $U_{rd} = 18.5$ becomes more or less constant for $U_{rd} = 18.5 – 32$. The $A_d^*$, however, remains small ($A_d^* = 0.03 – 0.12$) until $U_{rd} = 32$. The $A_u^*$ at $U_{rd} = 6.3 – 32$ is larger than $A_d^*$, while $A_d^*$ jumping to 0.53 at $U_{rd} \approx 33$ becomes larger than $A_u^*$. The $U_{rd} = 33$ can be considered as a critical reduced velocity $U_{rd,c}$ where $A_d^*$ jumps. The former phenomenon ($A_u^* > A_d^*$) is associated with the shear layer reattachment on the side or rear surface of the downstream cylinder, and the latter ($A_u^* < A_d^*$) is connected to the shear reattachment on the front surface of the downstream cylinder (Sun et al. [3]).

Similar trends of $A_u^*$ and $A_d^*$ are observed for $f_n^* < 1$, of course, with some differences (Fig. 2a, b). For instance, at $f_n^* = 0.8$ (Fig. 2b), $A_u^* > A_d^*$ at $U_{rd} = 5.3 – 23$; $A_u^* < A_d^*$ at $U_{rd} > 23$, and $A_d^*$ jumps at $U_{rd} = 25 = U_{rd,c}$, the features being qualitatively similar to those at $f_n^* = 1.0$. The difference is that the $A_u^*$ suddenly drops at $U_{rd} = 25.0$, and the vibration of the upstream cylinder is totally suppressed for $U_{rd} \geq 25.0$ ($A_u^* < 0.01$, Fig. 2b). On the other hand, the $A_u^*$ for $f_n^* = 1.0$ keeps boosting at $U_{rd} > 31.5$ ($A_u^* = 0.18 – 0.21$ for $U_{rd} > 32.0$). The suppression of the upstream cylinder vibration at larger $U_{rd}$ ($\geq 24$) is observed for $f_n^* = 0.6$ also (Fig. 2a), following a drastic drop at $U_{rd} = 24 = U_{rd,c}$. The value of $U_{rd,c}$ decreases with decreasing $f_n^*$, perhaps connected to the larger $A_d^*$ value before the jump/drop compared to $f_n^* = 1.0$ counterpart. When $f_n^* > 1.0$ (Fig. 2d, e, f), yet $A_u^* > A_d^*$ for smaller $U_{rd}$, and $A_u^* < A_d^*$ for larger $U_{rd}$, with $U_{rd,c} = 33, 36$ and 44 for $f_n^* = 1.2, 1.4$ and 1.6, respectively. Obviously, $U_{rd,c}$ postpones as $f_n^*$ increases, given that the values of $A_u^*$ and $A_d^*$ before the jump/drop do not change with $f_n^*$. Here, at $A_u^* > A_d^*$ regime, the $A_u^*$ is more or less constant for $f_n^* > 1.0$, and again $A_u^* = 0$ in $A_u^* > A_d^*$ regime. In $A_u^* > A_d^*$ regime, the $A_d^*$ is very small except for the small peak in $A_d^*$. The small peak in $A_d^*$ appears for all $f_n^*$ value at $U_{ru} = 11$ and/or 17, respectively, which is ascribed to the second and third harmonic excitations associated with the upstream-cylinder shear layer instability.

**FIGURE 2: DEPENDENCE OF UPSTREAM AND DOWNSTREAM CYLINDER VIBRATION RESPONSES ON $U_{ca}$ AND $U_{rd}$ AT (a) $f_n^* = 0.6$, (b) 0.8, (c) 1.0, (d) 1.2, (e) 1.4 AND (f) 1.6. $L/D = 1.5$.**
Figure 3 summarizes variations in $f_{o,u}$/$f_{n,u}$, $f_{o,d}$/$f_{n,d}$ and $f_{v}$/$f_{n,d}$ with $U_{r,u}$ and $U_{r,d}$ at three different $f^*_{n} = 0.8$, $1.0$ and $1.2$, where $f_{o,u}$ and $f_{o,d}$ are the upstream and downstream cylinder oscillation frequencies, respectively, and $f_{v}$ is the vortex shedding frequency. An isolated cylinder Strouhal number $St (= 0.2)$ line is added in Fig. 3(c, f, i) for a comparison purpose. The vertical dashed lines are used to separate each regime. The domain can be divided into three regimes following Fig. 2, namely, no lock-in (0), vortex excitation (VE), and galloping (G) regimes. The galloping regime is further subdivided into $G_1$, $G_2$, $G_3$ and $G_4$ regimes. Regime $G_1$ is characterized by a large $A^*_u$ and small $A^*_d$, while regime $G_2$ is featured by $A^*_d = 0$. Regime $G_3$, on the other hand, displays $A^*_u = 0$ and very large $A^*_d$. Very large $A^*_d$ and small $A^*_u$ appear in regime $G_4$. For $f^*_{n} > 1.0$, both upstream and downstream cylinders vibrate at their natural frequency (Fig. 3d, e). The vortex shedding frequency locks-in with $f_{n,d}$ ($U_{r,d} = 5.3 – 11.4$), $2f_{n,d}$ ($U_{r,d} = 13.7 – 16.1$) and $3f_{n,d}$ ($U_{r,d} = 18.5 – 32$) where $A^*_u > A^*_d$, and with $5f_{n,d}$ at $U_{r,d} > 32$ where $A^*_u < A^*_d$. Note that a peak at $f_{n,d}$ can always be detected which is connected to the cylinder vibration. For $f^*_{n} > 0 < 1.0$, in the galloping vibration regime, the vortex shedding frequency is dominated by $f_{n,u}$ and its higher harmonics for $A^*_u > A^*_d$, and then by $5f_{n,d}$ when $A^*_u < A^*_d$ (Fig. 3c, i). When $A^*_u > A^*_d$ and $A^*_d = 0$, both $f_{n,u}$ and $f_{n,d}$ can be detected in the downstream cylinder displacement (Fig. 3b, h). On the other side, the same occurs for the upstream cylinder as $A^*_u < A^*_d$ and $A^*_u = 0$ (Fig. 3a, g). That is, the cylinder vibrating at a larger amplitude at the two frequencies may suppress the vibration of the other cylinder. It may, therefore, be deduced that the galloping vibration suppression of the upstream or downstream cylinder during the galloping vibration of the other cylinder is connected to the difference between $f_{n,u}$ and $f_{n,d}$.

**Vibration and frequency response characteristics at $L/D = 2.0$**

The dependence of $A^*_u$ and $A^*_d$ on $U_{r,u}$ or $U_{r,d}$ and $f^*_{n}$ at $L/D = 2.0$ is shown in Fig. 4. At $f^*_{n} = 1.0$ (Fig. 4c), both cylinders experience VE at $U_{r,d} = 5.3 – 9.3$, which is followed by $A^*_u$ and $A^*_d = 0$ at $U_{r,d} = 10.3 – 18$. The upstream cylinder undergoes a small amplitude vibration at $U_{r,d} > 18$, while the downstream

![FIGURE 3: VARIATIONS IN $f_{o,u}$/$f_{n,u}$, $f_{o,d}$/$f_{n,d}$ AND $f_{v}$/$f_{n,d}$ WITH $U_{r,u}$ AND $U_{r,d}$: (a, b, c) $f^*_{n} = 0.8$, (d, e, f) $f^*_{n} = 1.0$ AND (g, h, i) $f^*_{n} = 1.2$. $L/D = 1.5$.](image)
cylinder first vibrates with a small amplitude for $18 < U_{r,d} < 29$ and with a large amplitude for $U_{r,d} > 29$, a jump in $A_d^*$ occurring at $U_{r,d} = 29 = U_{r,d,c}$. Obviously, the galloping and VE are separated.

For $f_n^* < 1.0$ (Fig. 4a, b), VE is suppressed for the downstream cylinder. In the galloping vibration regime, the vibration amplitude of the upstream cylinder is reduced compared to $f_n^* = 1.0$ counterpart, while that of the downstream cylinder remains unchanged. At $U_{r,d} > U_{r,d,c}$, the downstream cylinder vibrates violently ($A_d^* = 0.74 - 0.75$), and the upstream cylinder vibration is totally suppressed for both $f_n^* = 0.6$ and 0.8. With regard to $f_n^* > 1.0$ (Fig. 4d, e, f), the same vibration characteristic is observed for the downstream cylinder, and the upstream cylinder vibration is completely suppressed, regardless of smaller or larger $A_d^*$. The maximum $A_u^*$ in VE regime lies in 0.15-0.20 depending on $f_n^*$. To sum up, as $f_n^* < 1.0$ or $> 1.0$, in comparison with $f_n^* = 1.0$ where both cylinders undergo VE, the upstream cylinder still experiences VE while no VE is identified for the downstream cylinder. In addition, in the galloping vibration regime, the vibration of the upstream cylinder appearing at $f_n^* = 1.0$, seems to be reduced/suppressed when $f_n^* < 1.0$ or $f_n^*> 1.0$.

Figure 5 summarizes variations in $f_{o,d}/f_{n,d}$, $f_{o,d}/f_{n,d}$ and $f_{o,d}/f_{n,d}$ with $U_{r,u}$ and $U_{r,d}$ of $L/D = 2.0$ at three different $f_n^* = 0.8$, 1.0 and 1.2. Note that an $A_u^* < A_d^*$ relationship prevails in $G_1$ and $G_2$ regime, as opposed to $A_u^* > A_d^*$ for $L/D = 1.5$. For $f_n^* = 1.0$, both cylinders vibrate at their natural frequency (Fig. 5d, e). The vortex shedding frequency during the galloping vibration regime first follows $St \approx 0.17$ at smaller $U_{r,d}$ ($A_d^* < 0.2$ in Fig. 4c) and then locks-into $5f_{o,d}$ at larger $U_{r,d}$ ($A_d^* > 0.7$ in Fig. 4c). The smaller $St \approx (0.17)$ compared with that of an isolated cylinder ($St \approx 0.2$) is connected to the reattachment of the upstream cylinder shear layer on the front surface of the downstream cylinder; a reattachment flow corresponds to a smaller $St$ than the single cylinder flow [4]. For $f_n^* < 1.0$ or $> 1.0$, when VE happens, the vortex shedding frequency locks-into $f_{n,u}$ (Fig. 5c, i). Both $f_{n,u}$ and $f_{n,d}$ can be observed for $f_{o,d}$ (Fig. 5b, h) where no vibration occurs. In the galloping vibration regime, except for the frequency following $St = 0.17$, $f_{o,d}$ and $2f_{o,d}$ can also be observed as the downstream cylinder vibrates violently (Fig. 5c, i). It may be inferred that the wake flow is mainly governed by the downstream cylinder whose vibration is larger than the upstream cylinder’s. Besides, as the upstream cylinder vibration is inhibited, $f_{n,u}$ have two values at $f_{n,u}$ and $f_{n,d}$ (Fig. 5a, g), which may indicate that the vibration suppression is connected with $f_{n,d}$ different from $f_{n,u}$.

CONCLUSIONS

Flow-induced vibrations of two tandem circular cylinders of different natural frequencies are investigated for frequency ratios $f_n^* = 0.6, 0.8, 1.0, 1.2, 1.4$, and $1.6$ at $L/D = 1.5$ and 2.0. The focus is given on how the change in $f_n^*$ influences the vibration amplitude, frequency response, and vortex shedding frequency. At $L/D = 1.5$, vibration characteristics of either cylinder are highly dependent on $f_n^*$ and $U_{r,d}$. Based on the dependence of the vibration characteristics on $f_n^*$ and $U_{r,d}$, three regimes are identified, e.g., no lock-in (0), vortex excitation (VE), and galloping (G) regimes. The galloping regime itself reflects
different facets depending on $f_n^*$ and $U_{rd}$. As such, the galloping regime is further subdivided into $G_1$, $G_2$, $G_3$ and $G_4$ regimes. With an increase in $U_{rd}$ in the galloping regime, a drastic jump in $A_d^*$ and a drop in $A_u^*$ occur at $U_{rd,c}$. Connected to a lock-in of the shedding frequency with $f_{n,d}$. The galloping vibration of the upstream cylinder is suppressed for $f_n^* \neq 1.0$ at $U_{rd} > U_{rd,c}$, while that of the downstream cylinder is reduced/suppressed for $f_n^* > 1.0$ at $U_{rd} < U_{rd,c}$.

Similar vibration response regimes are identified for $L/D = 2.0$, with some distinct behaviors. VE and galloping vibration regimes are separated by a no-vibration regime. The galloping vibration of the upstream cylinder is suppressed for conditions $f_n^* \neq 1.0$ at $U_{rd} > U_{rd,c}$ and $f_n^* > 1.0$ at $U_{rd} < U_{rd,c}$. When the vibration of a cylinder is suppressed, two peaks (one at the cylinder own natural frequency and another at the other cylinder natural frequency) can be found at the power spectrum of the cylinder displacement. When the vibration of a cylinder is suppressed, two frequencies (one at the cylinder own natural frequency and another at the other cylinder natural frequency) can be found for the cylinder displacement.

The downstream-cylinder vortex shedding frequency is dominated by $f_{n,u}$ and its higher harmonics for $A_u^* > A_d^*$. The wake flow is dominated by the oscillation frequency of the cylinder whose amplitude is larger than the other.

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