A MODEL FOR FLUIDELASTIC INSTABILITY IN TUBE BUNDLES SUBJECTED TO TWO PHASE FLOW

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ABSTRACT

This study presents an attempt to model FEI in two phase flow in a normal square bundle. The model focuses on the bubbly flow regime of two phase flow. The flow around the tube is idealized as one dimensional flow in channels and is composed of two phases: continuous phase and dispersed phase. The dispersed phase is accounted for by spherical bubbles. The motion of each individual bubble was modelled including bubble-to-bubble interaction, break-up, coalescence. By tracking the bubbles in the channels, it was possible to calculate the change in the flow density around the tube. Prediction of the stability threshold showed a very promising results when compared with the experimental data.

1 Introduction

Flow-Induced Vibrations are a concern in the design and the operation of nuclear steam generators. The excitation mechanisms can be categorized as turbulence, periodicity, and fluidelastic instability (FEI). Fluidelastic instability has received the greatest attention because of the devastating effect on the tube integrity when this mechanism prevails. The damage is manifested in the form of fretting wear at the support locations or at mid-spans where tube-to-tube clashing takes place. Of major interest are the tubes in the U-bend region where the tubes are subjected to a crossflow of steam and water mixture. These damages can cause serious failures and often result in considerable cost. Because of their devastating effects, FEI was the subject of intensive investigation on both the experimental and modelling fronts.

One of the earliest efforts that dealt with FEI is the work of Roberts [1]. Subsequently, Connors [2] and Blevins [3] developed a quasi-static model that resulted in an identical expression to Roberts’. This expression relates the critical flow velocity to the mass-damping parameter through two constants. Tanaka and Takahara [4] developed an unsteady flow model where they assumed that the fluid forces acting on a cylinder are the result of the cylinder motion and the motion of the neighbouring cylinders. These forces were expressed in terms of force coefficients and phases. A series of precise and difficult experiments is required in order to obtain the force coefficients. Chen [5] re-derived the same model in terms of fluid-added inertia, damping, and stiffness components. Using a quasi-steady approach Price and Paidoussis [6] devised a model that introduced a time lag between the fluid forces and the cylinder motion. Lever and Weaver [7] proposed a semi-analytical model that utilized basic fluid mechanics coupled with the tube motion to predict the onset of the stability. Later, Hassan et al. [9–11] extended this model for time domain simulations. In a series of papers, Hassan and Weaver [12–14]...
extended the same model to simulate the streamwise instability of tube bundles. All of these models were able to predict the stability threshold for tube bundles with good agreement with experimental data. However these models were developed essentially for single phase flows. Although, nuclear steam generators operate in two-phase flow, there have been very few attempts to modify the existing models to handle two-phase flow. The simplest way to adopt the single-phase models to account for the two-phase flow is the use of Homogeneous Equilibrium Model (HEM). Such approach were used extensively utilizing the Connors equation (Pettigrew [19]). However, these models assume the same velocity for both gas and liquid phases. In the real vertical two-phase flow there is a slip between the motion of the two phases. Other attempts were directed at measuring fluid forces in two phase flow. For example, Mureithi et al. [15] and Inada et al. [16] measured the fluid force coefficients in two-phase steam-water flow in order to utilize them in the unsteady flow model. Their results showed weak effect of the surrounding tubes which made it difficult to utilize the full model accounting for a fully flexible tube bundle. Recently, Shahriary et al. [17] measured the force coefficients required for the quasi-steady model in the case of air-water mixture.

Using the Computational fluid dynamics technique (CFD), Sadek et al. [18] developed a model to predict the fluid forces acting on a cluster at tube subjected to two-phase flow. The predicted force coefficients were then used in an unsteady flow model to predict the stability threshold. The predicted stability threshold agrees well with the experimental counterparts.

In this paper, a model for bubbly flows is developed. The framework will utilize the channel flow model to represent the continuous phase while presenting the dispersed phase by a number of bubbles. The bubbles instantaneous position, size and velocity are predicted by considering all forces acting on individual bubbles. The capability of the numerical approach in simulating bubbly flows is tested by comparing its stability threshold prediction against the experimental data.

2 Fluidelastic Instability Model

2.1 Tube Bundle Model

The dynamic equation of motion of a tube can be presented as:

\[ M\ddot{x} + C\dot{x} + Kx = F_{FEI} \]  

(1)

where \( M, C \) and \( K \) are the effective mass, damping, and stiffness, respectively. \( F_{FEI} \) is the fluidelastic instability force due to the tube motion, which can be calculated by integrating the pressure around the tube between the attachment and the separation points.

2.2 Continuous Phase Model

Similar to the work by Lever and Weaver [8], the flow inside the bundle is idealized by a series of flow channels, as shown in Fig. (1). The flow through the channel is assumed to be one-dimensional, incompressible, and inviscid. The location at any point along the channel is described by coordinate \( s \). The channel area is assumed to have a steady state term, function of the position \( A(s) \) and a transient term, \( a(s,t) \). The transient term (perturbation) is caused by the tube motion. The channel perturbation between the attachment \( s = -s_a \) and separation \( s = s_s \) is equal to the tube displacement. However, a time lag between the tube displacement and the area perturbation takes place in both upstream and downstream the channel. The time lag is inversely proportional to the flow velocity and directly proportional to the location \( s \) along the flow channel.

The above described model was primarily applied to single phase flow [9–11]. In the current work, the above described formulation was adopted to model the continuous phase of the two-phase flow. The gas content in the continuous phase is described by the void fraction of \( (\alpha) \). As such a formulation on the form of volume-weighted equations are applied to the carrier phase. In other words, the fluid was considered compressible and Navier-Stokes are solved using the local mixture density at any location along the streamtube. The unsteady continuity equation was used to calculate the velocity field in the channels. Once the velocity field was obtained, the unsteady momentum equation was used to find the pressure along the
channel. The pressure was integrated along the tube channel interface to obtain the unsteady fluid force.

### 2.3 Dispersed phase

The dispersed phase contains a large number of entities (bubbles). Each bubble was modelled as a sphere and its motion were individually tracked. It is possible to predict the motion of each bubble by applying Newton’s second law. The total force acting on the bubble $\vec{F}$ could be subdivided into a number of components:

$$
\vec{F} = \vec{F}_L + \vec{F}_D + \vec{F}_b + \vec{F}_i
$$

where $\vec{F}_L$, $\vec{F}_D$, $\vec{F}_b$, and $\vec{F}_i$ are the lift, drag, buoyancy and impact force, respectively.

The lift force accounts for the effect caused by the vortex formation behind the bubble moving at a velocity of $U_g$ in a liquid flowing at a velocity of $U_l$. This force can be expressed as equation (3).

$$
F_L = C_L(t) \frac{\rho_t \pi D_b^2}{2} (\bar{U}_g - \bar{U}_l) |\bar{U}_l| \left(\sin(2\pi f_1 + \phi_1) + \sin(2\pi f_2 + \phi_2)\right)
$$

The frequencies of the alternating vortices’s $f_1$ and $f_2$ were calculated from the two values of Strouhal number $St$ reported by Sakamoto et al. [22]. The drag force $F_D$ acting on any bubble is a function of the relative velocity components $(\bar{U}_g - \bar{U}_l)$ as well as the projected area to the flow. The direction of the drag force is opposite to the relative fluid velocity with respect to the bubble

$$
F_D = \frac{1}{2} C_D \rho_l \frac{\pi D_b^2}{4} (\bar{U}_g - \bar{U}_l) |(\bar{U}_g - \bar{U}_l)|
$$

The drag coefficient $C_D$ was the subject of numerous investigations. The expression of Ishii [23] relates $C_D$ to the vapour void fraction and Reynold’s number and was utilized in this work:

$$
C_D = \frac{24(1 + 0.1(Re(1 - \varepsilon))^{0.75})}{Re(1 - \alpha)}
$$

Bubble-to-bubble collision takes place when the distance between the centers at two adjacent bubbles is less than the sum of their radii. The collision force is calculated using the elastic contact model by conserving both the kinetic energy and momentum. In this model the overlap between two colliding bubbles is first calculated then the force required to remove the overlap is obtained.

The buoyancy force $F_b$ is an upward force exerted by the liquid against the bubble weight. It is equal to the weight of the displaced liquid:

$$
F_b = \rho_b \frac{4}{3} \pi R_b^3 (\rho_l - \rho_g)
$$

### 2.3.1 Bubble Coalescence and Breakup

#### Bubbles Coalescence:

Several Coalescence criteria were proposed such as [24]. In this work, the empirical correlation by Duineveld [25] which was later modified by Senez et al [21] was utilized. In the criterion, the coalescence occurs if a critical Webber number is exceeded:

$$
We = \frac{2R \rho_l (U_l - U_b)^2}{\sigma} < 6.6
$$

where $U_b$ is the bubble velocity

#### Bubbles Breakup:

Bubbles break occurs due to the interaction with liquid flow. The bubble is considered to be stable if:

$$
Bo + We < 9
$$

where $Bo$ and $We$ are the Bond and Webber numbers [21], respectively. Bond number is a function of the surface tension $\sigma$ and is defined as:

$$
Bo = \frac{4g(\rho_l - \rho_g)R^2}{\sigma}
$$

### 2.3.2 Bubble Deformation

For a bubbly flow regime subjected to pressure perturbations, bubble size will change. Rayleigh-Plesset equation describes the dynamic response of a gas bubble in liquid suffering pressure change. By solving Rayleigh-Plesset equation numerically, it is possible to account for the bubble deformation.

### 3 Model Implementation

The bubble model including collision, breakup and coalescence was implemented with the continuous phase model in the time domain simulation model developed
by Hassan and Hayder [9]. Bubbles were generated randomly at the inlet of the flow channels. The size distribution of the bubble follows the recommendation of Wallis [20]. Initially each bubble was assumed to move vertically at a velocity equal to the mean flow and with zero lateral velocity. As bubbles move upward lateral and vertical velocities develop as other effects such as collision, lift, and drag forces come into effect.

The tube was given an initial displacement and the tube was allowed to respond to the developing destabilizing fluid forces. By solving the continuity and momentum equations using the average density at each section of the channel, the velocity and the pressure are obtained. Using the current local velocity and pressure, bubble forces (lift, drag, impact, etc.) are calculated and used to predict the bubbles velocities, locations for the next time step. Integrating the pressure around the tube enables calculating the fluidelastic instability force $F_{FEI}$. By using this force in equation (1), the displacement of the tube is obtained for the next time step.

4 Results and Discussion

Tube vibration: The first set of simulations were conducted for air-water mixture at atmospheric pressure and room temperature. The purpose of this set of simulations was to validate the model. A tube array $P/D = 1.47$ and a tube diameter of $d = 12.7mm$, and a structural damping of 1% was simulated. These parameters are identical to those Pettigrew’s [19] work to facilitate comparison with the experimental result. The tube response was examined for amplitude decay or growth. For each flow velocity, the tubes are given a disturbance (initial displacement), and the transient response is obtained. Typical examples for the results obtained using the homogeneous density approach are shown in Fig. 2. These simulations were done for a void fraction of 25%. Four cases in which the flow velocity is below (a,b), nearly equal to (c), and beyond the stability threshold (d) are shown. These cases are ‘well behaved’ with a smooth amplitude decay or growth.

On the other hand, Fig. 3 shows the tube vibration at various flow velocities for the same void fraction (25%). As it appear from the figure, for flow velocities lower than $1.3\ m/s$ the response is stable while the velocity $1.3\ m/s$ is marginally stable. Higher velocities show clear instability.

By comparing the decay trend in figures 2 and 3, We can notice that the current model was able to capture the two phase flow behaviour. While the fluidelastic force is the only force considered in this simulation, the response decay is not smooth. The response seems to have some random element and modulation. Similar time trace behaviour was reported by Moran [26].

FIGURE 2: Tube response for 25% void fraction at various flow velocity using the homogeneous density model

FIGURE 3: Tube response for 25% void fraction at various flow velocity

FEI Threshold (Critical flow Velocity): Figure 4 compares the stability threshold obtained using the current model with the experimental results reported by Pettigrew [19]. The stability threshold obtained by the current model was the average of three simulations to ensure repeatability. The difference between the three simulations did not exceed 5% except at 32% void fraction where the difference reached 12% of the average. The figure also shows the stability threshold obtained by the
FIGURE 4: Comparison between the results obtained from the current model with experiments [19]

HEM model. In this figure, the mass damping parameter MDP was calculated using the structure damping in air $\delta_s$.

**Slip:** The slip model, introduced by Feenstra et. al. [27], provides a qualitative mean to calculate the slip ratio $S$ (the ratio between the gas velocity and the liquid velocity). The slip $S$ was found to be a function of Capillary $Ca$ and Richardson $Ri$ numbers. The slip model equations are given by:

$$S = 1 + 25.7 (Ri \times Ca)^{0.5} \left( \frac{P}{D} \right)^{-1}$$

$$\alpha = \left[ 1 + S \frac{\rho_l}{\rho_g} \left( \frac{1}{x} - 1 \right) \right]^{-1}$$

Using the slip calculated by the above equations, a value for the slip $S$ was calculated for the same experimental conditions reported by Pettigrew [19] and compared with the slip ratio obtained from the simulations. The slip ratio obtained from the simulation was calculated as a mathematical average of the ratio between the bubbles velocity to the liquid velocity. Figure (5) shows a comparison of the slip factor predicted by the current model and Feenstra’s model [27]. The comparison shows good agreement between the current work and Feenstra’s model.

**Effect of Density ratio:** A study was performed to investigate the effect of the density ratio ($\rho_L/\rho_g$) on the stability threshold using the current model. In this study, the liquid density was kept constant at 1000 kg/m$^3$ while the gas density was adjusted to produce the desired density ratio. Figure 6 shows the predicted stability at various density ratios. As seen from the figure, the density ratio shows some effect on the stability threshold especially at $MDP = 0.23$ and 0.26. At higher mass damping parameter, this effect diminishes back which does not agree with some of the literature data. Further investigations are needed to explain this behaviour.

5 Conclusion

A model to predict FEI threshold of two-phase flow in normal square tube bundles was developed. The framework includes modelling the structural motion and the two phase-flow. In the two phase model, the continuous phase was presented by a simplified 1-D, incompressible and inviscid flow, while the disperse phase was represented by a number of bubbles. The motion at each bubble was tracked and the instantaneous distribution of the dispersed phase was obtained. The local variable den-
sity along with the flow perturbation due to the tube motion were used to calculate the flow field including the unsteady flow forces. The model was utilized to predict the FEI in a tube array. The predicated stability threshold compares very well with the available experimental data for void fractions up to 35%. The results are very promising as the model represents a step towards a realistic model for two phase flow in tube arrays. Model expansion to simulate higher void fractions is planned.

REFERENCES


