THE NUMERICAL STUDY OF THE WAVE REFRIGERATOR WITH ROTATING GAS DISTRIBUTOR

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ABSTRACT

Shock wave refrigerators (SWR) refrigerate the inflow gas through periodical expansion waves with a rotating distributor. Due to its simple structure and robustness, SWR may have many potential applications if the efficiency becomes competitive with existing alternative devices. In order to improve the refrigerating efficiency, the characteristics of wave propagation and oscillation in the SWR are studied theoretically and numerically. Based on the shock tube theory, a simplified model is suggested, which includes the assumptions of flux-equilibrium and conservation of the free energy. This model allows the independent analysis of the operation parameters and design specifics. In order to analysis deeply the influence factors of the SWR efficiency, CFD is used to simulate the propagation of the vortex induced by the inflow injecting inside the oscillation tube. The reflection wave inside the tube with different shape of the nozzles and different structure of the tube tail are studied. The considerations to improve the SWR efficiency are given.

Key words: shock waves, refrigeration, expansion waves, rotation

INTRODUCTION

The Shock Wave Refrigerator (SWR) is a wave machine which refrigerates a gas source by the expansion waves. The conceptual illustration is shown on Fig. 1.

Fig. 1 Principle illustration of Pressure Wave Refrigerator

The over-pressure inflow is refrigerated at the cost of lowering its inner enthalpy. Meanwhile, the
transferred energy is used to heat the medium (same as inflow gases) inside the machine. Therefore, it is used as a heater as well as a refrigerator. In generally, there are two different models for SWR. One is static SWR and the other one is the rotating SWR. For the static SWR, because of the machine structure, the mixing between the inflow and outflow is so strong that the efficiency of the refrigerator is very limited, researcher turned to develop the rotating SWR.

The rotating SWR is consisted of a high-rate rotating distributor with nozzles and stationary expansion tubes which mounted around the distributor; the design concept of a typical rotating SWR is shown on Fig. 2. The over-pressure gas is filled in the rotating distributor, and then sprays into the expansion tubes when the tubes are overlapping with the nozzles. Cooled by the expansion waves, the gas is evacuated from receiver. Unlike the continuous expansion waves in a static SWR, the periodic expansion waves are generated by the rotating nozzles while the inflow and outflow are divided by the different phases during the rotating SWR operation. Therefore, it is advantage in structure comparing to the static SWR. Due to its simple structure and robustness, rotating SWR is expected to have numerous potential applications in industry and consumer products once its efficiency becomes competitive with alternative devices.

This way to use gas refrigeration was first tried by Power Jets Ltd in the early 1960's. Although a practical machine was not developed, some theoretical and experimental results were accumulated. In the early 1970s, the first commercial SWR was developed by two French companies (ELF Co. and Bertin Co.). In the following years, most of the studies were focused on the gas distributor structure instead of the inside gasdynamic mechanism. Until end of 1980s, researchers began to study the wave motion mechanism inside the SWR machine. Some experimental investigations on rotating SWR were carried out by several researchers, Marchal 1985, Yu 1989, Fang 1991, in which, some ways to increase the SWR efficiency were developed. In 1989, Yu proposed a simplified mode with shock wave theory to analysis the process of the rotating SWR. Meanwhile, numerical simulations for some special SWR were carried out by other researchers, Zehnder 1971, Zhang and So 1990, Saito et al. 1993. A typically SWR description and numerical investigation was presented by Galyukov et. al. (1994, 1996).

However, the flow mechanism inside the rotating SWR needs to be studied deeply because practical efficiency of rotating SWR is not competitive enough till now. For the flow mechanism study, lots of factors influence the efficiency of the rotating SWR, such as the rotational speed, length of the expansion tube, heat transfer condition, nozzle shape, and other specific designs; it was studied with theory analysis and numerical simulation in the paper.

2 THEROTICAL ANALYSES
2.1 SHOCK TUBE THEROTICAL WAY

In order to illustrate the basic operation principles of a typical SWR, an x-t wave diagram for one operational cycle is shown in Fig. 3 by a 1-D numerical computation, in which the overlapping of the gas distributor nozzle with expansion tube is considered as a transient process instead of gradual process.

According to the gas-dynamic analysis, four operational phases exist during one working cycle:
(1) Filling phase (the red arrowhead duration, $\lambda_1\tau$)
(2) Refrigerating phase (the first black duration along the T-axis, $\lambda_2\tau$ )
(3) Evacuating phase (the black arrowhead, $\lambda_3\tau$)
(4) Shutting phase (the top black duration along the Time-axis, $\lambda_4\tau$)

At the beginning, when the nozzles overlap with the expansion tube, a classical shock tube flow starts, a shock wave (or compressed waves) propagates downstream into the expansion tube, when the nozzle is shut by the distributor wall, a strong expansion waves are generated. The inflow is mainly refrigerated in this phase. In the third stage, the refrigerated gas flows into the receiver companying with a new expansion and some other reflection waves. In the last stage, the distributor nozzle is shut off again with a compressed wave propagating downward into the tube. After that, next cycle repeats again.

Suppose that the inflow is evacuated from the tube without being disturbed by any reflection waves and also the viscous drag is neglected during the SWR operation, the maximum ideal temperature reduction of the inflow gas can be deduced by 1-D isentropic flow theory. The formula is obtained in (2-1), where the superscript ‘o’ represents the total state and ‘a’ corresponds to the sonic speed.

$$\frac{\Delta T^o}{T^o_{in}} = 1 - \left(\frac{P_{out}}{P_{in}}\right)^{\frac{\gamma-1}{2}} - \frac{\gamma-1}{2} \left(\frac{u_{out}}{a_{in}}\right)^2$$  (2-1)

The formula is illustrated graphically in Fig. 4 with $\gamma=1.4$ at different $u_{out}/a_{in}$. The up-most curve is the limitation of $\Delta T^o$ with $u_{out}=0$. According to the equation (2-1), for a given pressure ratio ($P_{in}/P_{out}$), the ideal temperature reduction depends on the speed of outflow. The lower the outflow speed $u_{out}$, the higher the temperature reduction. In order to apply this conclusion, to increase the ratio of outflow duration to inflow duration in a rotating SWR design is a useful way.

For a practical rotating SWR, as the circular cross-section of the actual device will introduce additional 3-D effect when a nozzle is overlapping with an expansion tube, also strong heat conduct and viscous effect exist along the tube length, 2-D and 3-D numerical analysis are necessary to deeply study the mechanism of rotating SWR.

Fig. 4 Ideal temperature reduction in a perfect gas due to isentropic process

Typical density contours during the overlapping of the nozzle and tube are shown in Fig. 5. A vortex can be seen developing downward the expansion tube when the incident shock wave propagates.

However the 2-D Euler model is not enough to simulate the overlapping process. 3-D Navier-Stokes simulations would be required to fully capture the
flow, but these require excessive computing power making them impractical for the present investigation.

Based on the shock tube theory, thermodynamic theory and some reasonable simplifications for the processes of a rotating SWR, a 1-D analysis solution is proposed which is effective and expeditiously.

Fig. 6 Typical x-t diagram with density distribution along the expansion tube

Some substitution variables are defined here to be used in the analysis below.

\[
\alpha = \frac{T_1}{T_{in}} \quad \beta = \frac{P_1}{P_{out}} \quad \varepsilon = \frac{\lambda_3}{\lambda_1}
\]  

(2-2)

The possible flow x-t diagrams for optimal conditions are illustrated in Fig. 6. Based on the shock wave relationship and simple wave equations, the transformed momentum equation is shown in Eq. (2-3):

\[
\frac{P_{in}}{P_{out}} = \beta \left( \frac{2\gamma}{\gamma + 1} \frac{M_s^2}{\gamma - 1} \right) \left[ 1 - \frac{\gamma - 1}{\gamma + 1} \sqrt{\frac{M_s^2 - 1}{M_s^2}} \right] \frac{2\gamma}{\gamma + 1}
\]

(2-3)

The mass conservation equation is expressed in Eq. (2-4)

\[
\int_{\tau_0}^{\tau_e} \rho u d\tau + \int_{\tau_e}^{\tau_w} \rho u d\tau = 0
\]

(2-4)

Once the operation evolves steadily, the work done by the inflow source \((\Delta W)\) on the media gas is equivalent to the transferred heat which conducts to the ambient by the tube wall.

\[
\frac{K}{\gamma - 1} \left[ T_{in}^{\alpha} - T_{out}^{\alpha} \right] = \frac{N_{hub} \rho d}{Q \sigma} \int_{\tau_0}^{\tau_e} \kappa \left[ T - T_w \right] d\tau
\]

(2-5)

Here, \(N_{hub}\) is the number of working tubes; \(d\) is the diameter of the expansion tube; \(L\) is the length of the expansion tube; \(\sigma\) is the wall thickness of the expansion tube; \(\kappa^\prime\) is the thermal conductivity coefficient of the tube and \(T_w\) is the ambient temperature outside the expansion tube, \(Q\) is the flow flux in one operation cycle. Theoretically, the three initial variables, \(P_{in}/P_{out}\), \(\alpha\) and \(\beta\) can be solved by the three restriction equations (2-3), (2-4) and (2-5). All flow field parameters in Fig. 6 can be determined uniquely. Hence, the effects of operational parameters \(\alpha\), \(\beta\) and the special design \(\varepsilon\) for a certain rotating SWR (\(N_{hub}\), \(d\), \(L\), \(\sigma\), \(\kappa^\prime\)) can be analyzed independently.

### 3 RESULTS AND DISCUSSIONS

The ideal temperature reduction \(\Delta T^o/T_{in}\) at different \(P_{in}/P_{out}\) and \(\varepsilon\) are shown in Fig. 7, where \(\alpha\) is fixed at 1.25 and \(\varepsilon\) is given as 0.5, 1, 2 and 4, respectively. The uppermost curve represents the maximum of \(\Delta T^o/T_{in}\). The ideal \(\Delta T^o/T_{in}\) is increasing with \(\beta\) if \(\beta > 1\). However, a maximum \(\Delta T^o\) occurs if \(\varepsilon\) is less than 1, it means the flow is choked in this case. On the other hand, if \(\varepsilon\) is more than 4, the \(\Delta T^o/T_{in}\) is very close to the uppermost curve. Within a practical application range, The suitable choice for \(\varepsilon\) is about 4, which is agrees well with the other researchers’ experimental conclusion.

Usually, the effect of the spin rate \(\omega\) is related to the length of tube \(L\). The optimum matching condition depends on the timing \(\tau_0\) when the reflected shock wave returns to the exit of expansion tube. Hence, a long tube corresponds to a low \(\omega_{opt}\). However, the \(\Delta T^o\) of outflow will be insensitive to
the spin rate \( \omega \) if some accessory setups, such as damping tank, ring or orifice, are attached at the end of tube for eliminating the reflected waves, such a set-up is recommended.

Based on above equations (2-3) \( \sim \) (2-4), the average pressure inside the tube "\( \beta \)" is only determined by \( \varepsilon \) and \( P_{in}/P_{out} \), and it is nothing to do with the "\( \alpha \)". According to equation (2-5), a thin tube wall (small "\( \sigma \)"") with a good conductive material (large "\( \kappa \)"") is useful to lower the \( T_1 \), where \( T_1 \) is the average temperature inside the expansion tube when the operation is steadily. The number of expansion tubes "\( N_{ub} \)" affects the \( T_1 \), more the "\( N_{ub} \)", lower the \( T_1 \).

4 CONCLUSIONS

All of the analysis is based on an ideal 1-D model without the consideration of unsteady process and viscous effects. For a practical rotating SWR, although some 3-D effects like overlapping of the nozzle and the tube affects the efficiency of a SWR with a fine way, the analytical solution shows an expedient way to catch the general character of the rotating SWR.

1) For a given \( P_{in}/P_{out} \), The bigger, the ratio of the evacuation duration to inlet duration, the higher, the temperature reduction of rotation SWR.
2) Different material is suggested to be used in the expansion tube. The part near the inlet nozzles is made of insulator, such as plastic. The other part is made of good conductor of heat, such as copper.

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