ACOUSTIC RESPONSE OF MULTIPLE SHALLOW CAVITIES: EFFECT OF SEPARATION DISTANCE

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ABSTRACT
Flow over ducted shallow cavities can excite fluid resonant oscillations. These resonances often generate noisy environment at the work place and may eventually lead to structure failure of associated equipment. A common industrial application is the flow in corrugated pipes that can be modelled as a series of consecutive shallow cavities. In the current study, the effect of the separation distance on the aeroacoustic source of multiple shallow cavities is investigated. The source is measured using the Standing Wave Method (SWM) where two sets of three microphones are equipped, upstream and downstream of the cavities section, to measure the discontinuity in the acoustic pressure generated by the flow over the cavities. Two and three-cavity configurations are tested with a separation-to-cavity length ratio in a practical range from 0.5 to 1.375. At low and intermediate sound levels, extremum spacing ratios of 0.5 and 1.375 show constructive interference with high source values. However, at high excitation levels, 10% and higher, the source strength becomes inversely proportional to the separation distance. These findings are observed in both the double and triple-cavity configurations. On the other hand, the destructive interference spacing ratio is shown to be a function of the number of cavities.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$S$</td>
<td>Normalized aeroacoustic source</td>
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<tr>
<td>$St$</td>
<td>Strouhal number based on the cavity length</td>
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<tr>
<td>$U$</td>
<td>Flow velocity (m/s)</td>
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<tr>
<td>$U_{conv}$</td>
<td>Convection velocity of the vortical structures along the cavity shear layer (m/s)</td>
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<tr>
<td>$v$</td>
<td>Acoustic particle velocity (m/s)</td>
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<tr>
<td>$\rho$</td>
<td>Air density (kg/m$^3$)</td>
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<tr>
<td>$\tau$</td>
<td>Periodic time (s)</td>
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INTRODUCTION
Corrugated pipes are commonly used in offshore oil and gas fields. The corrugation geometry offers advantageous global flexibility. However, the shear layers that form over the corrugations are vulnerable to hydrodynamic instabilities. An acoustic standing wave can provide a feedback mechanism which sustains the initial hydrodynamic disturbance causing a fluid resonant oscillation. Belfroid et al. [1] reported acoustic pulsations with amplitude in the order of bars for pressurized corrugated pipes. This high level of pulsation can endanger the safety of the system operation.

Research interest has been dedicated to investigating the sound generation by the flow over a single cavity to establish a basic understanding of the phenomenon. Experimental approach based on analyzing the standing wave was proposed by Mohamed et al. [2] for a single cavity. The measurement technique was further adapted for the flow over multiple cavities by Shaaban and Ziada [3]. Another experimental approach relies on flow visualization using Particle Image Velocimetry (PIV) and applying Howe’s analogy [4] to compute the aeroacoustic power. This measurement technique was used by Finnegan et al. [5] to
evaluate the source for flow around two tandem cylinders and by Mohamed and Ziada [6] for a single shallow cavity.

Numerical simulations have also been extensively used to model the flow over a single cavity. Nakiboglu et al. [7] developed a CFD laminar flow model for a single shallow cavity. The model fairly predicted the resonance flow conditions but overestimated the acoustic power generated. In a subsequent study, Golliard et al. [8] improved the model performance by considering turbulence using k-ω model. More recently, Rajavel and Parasad [9] suggested a Large Eddy Simulation (LES) turbulence CFD model where the solver included the acoustics based on Ffowcs Williams and Hawkins (FW-H) analogy [10].

Limited research effort has dealt with the flow over multiple cavities. Understanding the aeroacoustic response of multiple cavities seems to be a first step towards comprehending long corrugated pipe fluid resonant oscillations. Nakiboglu and Hirschberg [11] investigated the self-excited oscillations of a two-cavity configuration supplemented by CFD modelling. They found the spacing ratio to affect the amplitude of self-excited oscillations, which seems to suggest different hydrodynamic interference patterns depending on the spacing ratio.

The aim of the current study is to investigate the effect of the separation distance on the aeroacoustic source of axisymmetric multiple cavities. The source is measured using the Standing Wave Method (SWM) technique [2,3]. Both two and three-cavity configurations are tested. The test setup allows for assessing the effect of the acoustic excitation level. Eventually, the hydrodynamic interference is analytically interpreted based on disturbance convection speeds.

**EXPERIMENTAL SETUP**

The aeroacoustic source of the multiple cavity configurations is measured using the SWM. The acoustic standing wave needs two microphone measurements in order to be reconstructed. In the current test facility, three microphones are used and the two parameters of the wave equation are evaluated using the least squares method. Consequently, three microphones are installed at both upstream and downstream of the multiple cavity test section as depicted in Fig. 1. The discontinuity in sound wave acoustic pressure at the cavity (∆P) is used to compute the non-dimensional aeroacoustic source as defined by Eq. (1). The acoustic velocity should be maintained constant throughout the test section. The maximum variation of the acoustic velocity across the different tested configurations is 3%.

\[
S = \frac{\Delta P / (0.5 \cdot \rho U^2)}{v/\bar{U}}
\]  

(1)

The piping system used in the current test facility is 4" nominal diameter schedule 80. The upstream pipe length is long enough to ensure fully developed flow conditions. The mean flow velocity is measured using a pitot tube. A speaker is attached to a downstream side branch which is used to excite the standing acoustic wave. The speaker is controlled using a function generator and an amplifier. This facilitates exciting the sound wave at the resonance frequency of the system and tuning the excitation amplitude (v) up to a relatively high level of 20% of the flow velocity. The suction blower used downstream of the piping system is connected to a variable speed drive to achieve different flow velocities that cover the resonance lock-in range.

Both double and triple-cavity configurations are tested in the current investigation. The cavity geometry is kept constant with a length (L) of 2" and corresponding depth (H) of 1". The separation plateau distance to the cavity length ratio (Lp/L) varies from 0.5 to 1.375. For each case, the tested Strouhal number (St = fL/\bar{U}) ranges from 0.35 to 1.2. This covers the first two shear layer modes. At each Strouhal number, the effect of the excitation level on the source trends is assessed by varying the acoustic velocity ratio in the range from 0.002 up to 0.2.

**RESULTS AND DISCUSSION**

**Analysis for the Two-Cavity Configuration**

The aeroacoustic source characteristics obtained by using the SWM are presented in this section for the two-cavity configuration. The real part of the source will be the main focus as it reflects the actual acoustic energy transfer between the cavity section and the mean flow. Positive real source means that the cavities are actually generating acoustic energy and presents a potential resonant flow conditions during which the power generated counterbalances the system losses. The aeroacoustic source is a non-dimensional quantity as previously defined by Eq. (1). Positive real sources are typically observed in two Strouhal ranges corresponding to the first two shear layer modes of the cavities. The current discussion will be concerned with the first shear layer mode because it generates strong aeroacoustic source and is reported to cause industrial problems. The literature usually refers to this resonance mode as the fundamental mode and is typically centered about a Strouhal number of 0.6.

The source term corresponding to the fundamental mode of the
two-cavity configuration is plotted in Fig. 2 for different spacing ratios at a low excitation level of 0.5%. The lowest spacing ratio of 0.5 and the largest one of 1.375 are characterized by strong source values. These source values exceed double that of a single cavity with the same geometrical parameters previously investigated by Shaaban and Ziada [3]. This indicates a constructive hydrodynamic interference between the two cavities at these spacing ratios. On the other hand, minimum peak source is observed for the configuration with spacing ratio of 0.75. Thus, this ratio can be considered a destructive interference spacing ratio. The lock-in range for the smallest spacing ratio case is characterized by a single primary peak at a relatively low Strouhal number. As the spacing ratio increases, this primary peak fades out and a secondary peak appears at a higher Strouhal number and eventually dominates at the spacing ratio of 1.375. The aeroacoustic source trends presented so far agrees well with the self-excited pressure amplitude features reported by Nakiboglu and Hirschberg [11] for a two-cavity configuration.

Strouhal number of the peak source value decreases when the excitation level is increased. A similar phenomenon was reported by Ziada [12] based on a flow visualization study for closed side-branches. By means of PIV measurements of the vorticity field of a single shallow cavity, Mohamed and Ziada [13] reported similar effect for high excitation levels.

Analysis for the Three-Cavity Configuration
The source term corresponding to the fundamental mode of the three-cavity configuration is plotted in Fig. 4 for different spacing ratios at an excitation level of 1%. The main findings observed for the two-cavity configurations, at $v/U$ less than 0.05, is still persistent. Spacing ratios of 0.5 and 1.375 are still associated with high source values. However, the destructive interference is shifted to a spacing ratio of unity. Consistent constructive interference ratio can be considered as a favorable spacing ratio where the vorticity field of the upstream cavity positively impacts that of the downstream cavity. On the other hand, the destructive interference is a more complicated mechanism. The downstream cavity in the two-cavity configuration at a spacing ratio of 0.75 is likely suffering from a negative feedback from the vorticity field of the first cavity. Thus, it may not possess enough coherent vorticity to impact a third installed cavity downstream and consequently the destructive interference spacing ratio is shifted for the overall source of the three-cavity configuration to a new value.

The three-cavity configuration source results for an excitation level of 5% is shown in Fig. 5. Again, this excitation level exhibits a transitional phase before establishing a new behavior at high excitation levels. At acoustic velocity ratio of 0.1, the normalized peak source of both the two and three-cavity configurations is plotted vs. the spacing ratios in Fig. 6. The peak source for each cavity configuration is normalized by the source.

![Graph](image-url)
corresponding to the spacing ratio of 0.5. The behavior of the source at such a high excitation level is clear where the source decreases with the increase in the spacing ratio. At the largest spacing ratio of 1.375, the peak source is 0.66 and 0.6 of that corresponding to spacing ratio of 0.5 for the double and triple-cavity configurations, respectively. This may be attributed to the effect of the excitation level on the convection speeds of vortical structures previously discussed.

\[ St = \frac{f L}{U} = \frac{f L}{U_{conv}} \times \frac{U_{conv}}{U} \]  

The disturbance propagation in pipes is an interesting topic in fluid dynamics. It has been the concern of many studies aiming at understanding the phenomenon of transition from laminar to turbulence. In this section, attention is focused on the disturbance convection speed along the pipe. The disturbance equation, derived by Sexl [14], was analyzed by Gill using an order of magnitude analysis approach. The convection speed asymptotically attained the same value of the flow velocity at high Reynolds numbers. The same disturbance equation was solved numerically by Salwen and Grosch [15] estimating the ratio between the convection and flow velocities to be 0.98 at a Reynolds number of 5 x 10^4. Similar findings were confirmed by O'Sullivan and Breuer [16] using a general Chebyshev polynomial for the mean velocity profile instead of the simple Poiseuille parabolic one. At Reynolds number of 10^4, the ratio between the convection speed and the flow velocity was 0.97.

\[ St = \frac{f L}{U} = \frac{f L}{U_{conv}} \times \frac{U_{conv}}{U} \]  

The constructive interference is expected to occur at the spacing ratio where the disturbance in the pipe spacing travelling from
the upstream cavity arrives in phase with the vortical structure cycle along the downstream cavity shear layer, i.e. both having the same periodic time as shown by Eq. (3). A schematic of a multiple cavity configuration is shown in Fig. 7. Consequently, the theoretical constructive spacing ratio is estimated to be 1.67 as compared to 1.375 from the SWM results. Such deviation may be caused by the effect of turbulence and the existence of a standing wave on the value of the disturbances convection speed. Such effects have not been accounted for in the theoretical analysis.

\[ \frac{L_p}{U} = \frac{L}{0.6U} \rightarrow \frac{L_p}{L} = 1.67 \] (3b)

![Figure 7: SCHEMATIC OF A MULTIPLE CAVITY CONFIGURATION WITH THE RELEVANT GEOMETRICAL PARAMETERS](image)

The constructive interference observed at low spacing ratios may be attributed to almost direct communication between the neighbouring shear layers. On the other hand, as the acoustic velocity ratio reaches 0.1 and beyond, the effect of the acoustic wave on the aforementioned convection speeds is likely to be substantial causing the new trends previously observed in the SWM results. The decrease in the peak whistling Strouhal number, observed in the SWM data at high \( \nu/U \), indicates a corresponding decrease in the vortical structures convection speed and is consequently expected to increase the value of the constructive spacing ratio. Thus, at high excitation levels, the spacing ratio of 1.375 may be trapped in an extended destructive interference range.

CONCLUSIONS

The aeroacoustic source has been measured for two and three-cavity configurations having different separation distances using the Standing Wave Method (SWM). The separation distance is found to be a critical parameter with a big impact on the source value. For cases with an excitation level less than 5%, extremum spacing ratios of 0.5 and 1.375 are associated with high source values indicating constructive hydrodynamic interference between the neighbouring cavities. This behavior changes for excitation levels of 10% or higher, where the source is found to consistently decrease with the increasing spacing ratio. This may be attributed to the decrease of the vortical structures convection speed, which travel along the cavity mouth, at high acoustic velocity ratios. Excitation levels of 5% exhibits a transition phase between the two aforementioned behaviors. These findings are consistent for both the double and triple-cavity configurations. The destructive interference spacing ratio changes with the number of cavities encountered in the tested configurations indicating relatively complicated interaction mechanism. The different hydrodynamic interaction patterns are interpreted analytically based on the convection speeds of disturbances in the pipe spacing and along the cavity shear layer.

REFERENCES

