AEROELASTIC EFFECT CONSIDERING STRUCTURAL GEOMETRICAL NONLINEARITY

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ABSTRACT
In the paper, Simo’s geometrically exact rod was introduced for the aeroelastic investigation considering structural geometrical nonlinearity. Two aeroelastic solvers were developed for the FSI problems. Their only difference is the solution of fluid equation. One is to use Theodorsen’s theory to calculate aerodynamic loads and PK method to solve the flutter equation in frequency domain. Another is to use the computational fluid dynamics (CFD) for the solution of the fluid equation in time domain. For a high-aspect-ratio wing with the thrust and the mass of engine simplified as the lateral follower force and the centered mass, the structural and engine parametric influences on aeroelastic characteristics were analyzed by the both developed solvers.

NOMEANCLATURE

\( EI \), the structural bending stiffness modulus; 
\( GJ \), the structural torsional stiffness modulus; 
\( m \), the resultant force of rod section; 
\( n \), the resultant moment of rod section or normal vector of the interface; 
\( u \), the displacement of structural deformation; 
\( p \), the linear momentum of rod; 

\( P \), the fluid pressure; 
\( \lambda \), the interface loads; 
\( \tau \), the fluid viscous stress; 
\( \varphi(S) \), the centerline curve of rod; 
\( \Lambda \), the sectional rotation matrix of rod. 

\( \pi \), the angle momentum of rod;

1 INTRODUCTION
High-altitude, long-endurance aircrafts are being considered for long-term surveillance, environmental sensing and communication relay. Due to the requirements of high performance and light weight, these aircrafts are usually designed with high-aspect-ratio wings, which will have large structural deformation in flight. Linear structural theory fails to accurately analyze their aeroelastic characteristics. The aeroelastic analyzing method with the consideration of structural geometrical nonlinearity needs to be developed.
Aerelastic effects considering structural geometrical nonlinearity were firstly investigated by Tang and Dowell [1] who used the nonlinear beam theory and ONERA aerodynamic model. Hodges [2] put forward to the geometrically exact intrinsic beam model deduced by Hamilton principle, and the NATASHA software was developed for HALE aircrafts which can consider the structural geometrical nonlinearity and flight dynamics [3-6]. Recently, it has been extended to treat the pre-twisted beams under distributed follower forces [7-8].

Simo [8] constructed the theory of geometrically exact rod based on differential geometry, which divided the structural movement as two parts, one is the arbitrary movement of central axis of the rod, and another is the finite rotation of the rod section to describe the arbitrary large deformation, which can be solved by the classic Newmark time integrated method with finite element discretization [8-12]. In the paper, Simo’s geometrically exact rod is used for the aerelastic solution considering the structural geometrical nonlinearity.

In the paper, two aerelastic solvers are developed for FSI problems, in which Simo’s geometrically exact rod is used for Computational Structural Dynamics (CSD). For the aerodynamic loads, one is to use Theodorsen’s theory and solve the aerelastic equations in frequency domain, and another is to use CFD with time domain method.

For a high-aspect-ratio wing with the thrust and the mass of engine simplified as the lateral follower force and the centered mass, the both developed methods are used for the investigations on the parametric influences on aerelastic characteristics by the changes of the vertical bending-torsional stiffness ratio, centered mass, lateral follower force and the location of centered mass.

2 METHODOLOGIES

2.1 GEOMETRICALLY EXACT ROD

The kinematics description of geometrically exact rod can be written as

\[ Q = \{ \Phi = (\phi, \Lambda) | \phi: [0, L] \rightarrow R^3, \Lambda: [0, l] \rightarrow SO(3) \} \]  

In here, Q represents all possible configurations of the rod, \( \phi(S) \) the centerline curve of the rod and \( \Lambda(s) \) the sectional rotation matrix, as shown in figure 1.

The dynamic equations of geometrically exact rod can be written as

\[ \ddot{p} = n' + \tilde{n} \]  
\[ \ddot{n} = m' + \phi' \times n + \tilde{m} \]

In here, (\cdot)' represents the derivatives of arc length and (\cdot) the derivatives of time. \((p, n)\) are the linear and angle momentum. \((n, m)\) are the resultant force and the resultant momentum at the rod section , and \((\tilde{n}, \tilde{m})\) are the external force and momentum of unit length at the rod section.

The variation form of balance equation of the rod is

\[ G(\phi, \Lambda; \eta, v) = G_{int} + G_{dyn} - G_{ext} = 0 \]  

Where

\[ G_{int} = \int_0^L [n \cdot (\eta' - v \times \phi') + m \cdot v'] dS \]  
\[ G_{dyn} = \int_0^L (\ddot{p} \cdot \eta + \ddot{n} \cdot v) dS \]  
\[ G_{ext} = \int_0^L (\tilde{n} \cdot \eta + \tilde{m} \cdot v) dS \]

\[ \partial_{\Gamma} \] represents the boundary values

2.2 FSI COUPLING ALGORITHM

Fluid structure coupling conditions are essentially rooted in two fundamental facts of mechanics: kinetic and dynamic continuity which can be defined by

\[ u_f = u_s = u_{fr} \text{ on } \Gamma \]  
\[ \lambda_f + \lambda_s = 0 \text{ on } \Gamma \]

where \( u_f \) and \( u_s \) are displacements of fluid and structure on the interface, \( \lambda_f \) and \( \lambda_s \) are the interface loads of fluid and structure, respectively.

The fluid loads can be computed by CFD as

\[ \lambda_f = -P \cdot n_f + \tau \cdot \nu_f \]  

where \( P \) is the fluid pressure, \( \tau \) is the viscous stress, \( n_f \) is unit normal vector on the fluid interface.

We could define a Steklov-Poincaré operator for the fluid domain as follows

\[ \lambda_f = F(u_f) \]  

Similarly, \( \lambda_s \) on the interface for the structure domain could also be defined by

\[ \lambda_s = S(u_s) \]  

We define further an abstract operator \( G = S^{-1}(-F) \), then FSI coupling method can be transformed into the form of fixed point equation

\[ G(u_{fr}) = u_{fr} \]  

The equation can be solved with fixed point method with dynamic relaxation [13].
3 RESULTS AND DISCUSSIONS

3.1 COMPUTATIONAL MODEL

Taken HALE wing beam model [13] as an example, its structural parameters are shown in Table 1

3.2 STRUCTURAL MODAL INFLUENCES WITH THE ENGINE FOLLOWER THRUST

Due to the rectangle straight wing of HALE wing, strip theory can be used for the flutter analysis. It is enough that the wing was discretized by 32 finite elements and only considering the first eighth modals. Structural modals influences are investigated with the changes of the parameters such as the stiffness ratio of λ, the mass of engine, the follower thrust of engine and the installation position of engine. After solving the static equilibrium equation, the eigenvalue analyses can be done in the equilibrium position. For example, taken as λ = 2, the mass of engine $m_e = 0$ and only considering the horizontal follower thrust acting on the wing tip, the critical thrust can be calculated as 332.6N, which is agreement with the value of 335.1N of the reference [13]. The structural frequencies and damping versus the engine thrust are shown in figure 2, in which $V_i, H_i, T_i$ represent the vertical, horizontal and twisting modals, respectively. It is indicated that the first horizontal bending frequency is unchangeable with the thrust and the first vertical bending frequency and the first torsional frequency are increasing with the thrust, however, the second and third vertical bending frequencies are decreasing with the thrust. As the larger the critical thrust value is, the system become unstable.

<table>
<thead>
<tr>
<th>Table 1 HALE wing structural parameters</th>
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<tbody>
<tr>
<td>Half span length</td>
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<tr>
<td>Chord length</td>
</tr>
<tr>
<td>Airfoil</td>
</tr>
<tr>
<td>Mass per unit length</td>
</tr>
<tr>
<td>Moment of inertia</td>
</tr>
<tr>
<td>Flexible axis</td>
</tr>
<tr>
<td>Center of mass</td>
</tr>
<tr>
<td>Span bending stiffness $(EI_x)$</td>
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<tr>
<td>Chord bending stiffness $(EI_y)$</td>
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<tr>
<td>Torsional stiffness (GJ)</td>
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<tr>
<td>Air density</td>
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<tr>
<td>Flight altitude</td>
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</table>

Fig. 2 Structural frequencies and damping versus the engine thrust for $\lambda = 2, m_e = 0, y_e = 1$

The critical thrust with the spanwise positions of engine installation is shown in figure 3. It is indicated that the critical thrust is decreasing rapidly with the spanwise position.
3.3 AEROELASTIC CHARACTERISTICS OF HALE WING

Two FSI solvers are developed in the paper, in which the structural displacement is simulated by Simo’s geometrically exact rod, and the flow load is calculated by CFD and Theodorsen’s theory, respectively. For the above described HALE wing, the parameter influences on flutter are studied with the developed CFD/CSD and Theodorsen/CSD. The changeable tends of the both methods are agreeable, even the quantitative values have some differences.

Figure 4 shows the flutter velocities and frequencies versus the thrust. As the thrust is smaller, the flutter velocity is increasing with thrust, however, as the thrust is larger, the flutter is decreasing with the thrust. At p=3 the maximum of flutter velocity is obtained, at the same time, its responding frequency is very sensitive with the thrust. Figure 5 gives the flutter velocities and frequencies versus the spanwise position of engine installation. At 75% position of spanwise length, the maximum of flutter velocity can be obtained, and flutter frequency is decreasing with the spanwise position. Figure 6 gives the flutter velocities and frequencies versus the chordwise position of engine installation. As the engine is installed behind the flexible axis of the wing, flutter velocity decreases, and as the engine is installed forward the flexible axis, the flutter velocity increases.

Fig. 3 Critical thrusts versus the spanwise position of engine installation for $\lambda = 2, m_e = 0, y_e = 1$

Fig. 4 Flutter velocity and frequency for $\lambda = 2, y_e = \frac{15}{16}, x_e = 0, m_e = 0$

Fig. 5 Flutter velocity and frequency for $\lambda = 2, p = 2, x_e = 0, m_e = 0.2$
5 CONCLUSIONS

Simo’s geometrically exact rod was adopted for the simulation of structural geometrical nonlinearity. By coupling CFD and Theodorsen’s theory, respectively, both FSI solvers were developed in the paper. For HALE high-aspect-ratio wing, firstly, the influences of the modals on structural stiffness ratio, the position of engine installation and its size of thrust were investigated, then the flutter characteristics versus structural and engine parameters were analyzed.

(1) Comparing the geometrically exact intrinsic beam model developed by Hodges, Simo’s geometrically exact rod is more concisely and is also suitable for the analyses of structural geometrical nonlinear aeroelasticity.

(2) The structural parameters and engine installation position have large influences on flutter velocity and frequency with CFD/CSD coupling method and linear Theodorsen/CSD method, whose predict nearly the same results.

(3) In the next step, the coupling method of the Simo’s geometrically exact rod with CFD will be used for the transonic flutter simulation with engine follower force, which cannot simulated by the linear aerodynamic method.

ACKNOWLEDGMENTS

This work was supported by National Natural Science Fund of China(No. 11672303).

REFERENCES


Fig. 6 Flutter velocity and frequency for $\lambda = 2, y_e = 0, m_e = 0.2$
