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A 3D NON-LINEAR REDUCED ORDER MODEL FOR A CANTILEVERED PIPE CONVEYING FLUID UNDER VIV

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ABSTRACT

This paper proposes a three-dimensional non-linear reduced order model for a cantilevered pipe ejecting fluid under vortex-induced vibrations (VIV). The Modular Modeling Methodology which, in previous works by the authors, has proven to be sucessfull in the derivation of planar non-linear models for this system, is again applied, conciliating the use of first principles (a Hamiltonian formulation for the problem of a pipe ejecting fluid) along with a wake-oscillator phenomenological model for VIV. This wake-oscillator is based on van der Pol equation and was originally proposed for a rigid cylinder performing in-line and cross-wise oscillations with respect to the direction of the free-stream velocity of the external flow. Considering the weak three-dimensionality observed in the flow around a rigid cylinder, the two-dimensional wake-oscillator is assumed to provide a valid description of the interaction with the external flow, at each cross section of the pipe. Numerical simulations are performed for 4 selected scenarios already tested for a planar model of this system, concentrating on assessing the dynamic response in the neighborhood of a bifurcation of the pipe model and in the associated VIV lock-in peaks. In future work, this model shall be used for a more comprehensive study of the response of this system under various combinations of internal and external flows.

INTRODUCTION

This paper focuses on the derivation, applying the Modular Modeling Methodology (MMM) introduced by [1, 2], and preliminary analysis of a non-linear reduced order model for a cantilevered inextensible pipe ejecting fluid under vortex-induced vibrations (VIV). The aim of this work is to extend the model already derived in [3, 4], using the MMM, by allowing the pipe to perform a three-dimensional motion instead of being artificially constrained to a planar motion orthogonal to the direction of the free-stream external flow, a simplifying hypotheses adopted in these previous works. Such an extension involves not only proper modifications in the modeling of the pipe ejecting fluid, but also the adoption of a two-dimensional phenomenological model for VIV. It is not the intention of the present paper to carry out comparisons with possibly existing numerical results on the same problem, under the same boundary conditions. Neither is its intention to make comparisons with possibly existing experimental results. As a matter of fact, experimental results on the concomitant phenomena, cantilevered flexible pipes conveying fluid under VIV, could not be found in the technical literature. Such comparisons will be surely done, but will be left to a further paper. For now, the present work is meant to provide some theoretical insights on the concomitant phenomena, and how they could affect one another.

In previous papers, the Modular Modeling Methodology was also sucessfully applied for the derivation of non-linear finite element method (FEM) [5, 6] and reduced order [7, 8] models for the classical problem of a pipe conveying fluid, introduced by Benjamin [9], to which Païdoussis [10] wrote a comprehensive treatise, with an extensive literature review.

Following the same strategy adopted in the derivation of a planar model for this system [3, 4], vortex-induced vibrations are modelled according to the wake-oscillator concept, in which a non-linear equation (typically, a van der Pol equation) is adopted aiming at emulating the vortex-wake dynamics. Such an approach was well developed for the study of VIV in rigid cylinders and deals both with the cases in which it is constrained to oscillate only in the cross-flow direction [11,12] and in the in-line and cross-flow directions simultaneously [13,14].

Recently, the combined effects of VIV and internal flow have been investigated. References [15–17] investigate the response of pinned-pinned tubes pipes conveying fluid subjected to simultaneous horizontal top-motion excitation and VIV. In those works, only cross-wise oscillations are allowed and the top-motion excitation significantly affects the structural oscillations. In [16], a flexible cylinder constrained to oscillate only in the cross-wise direction and concomitantly subjected to a pulsating internal flow and VIV was studied using perturbation methods. Reference [18] also addresses the pinned-pinned pipe conveying fluid; however, simultaneous displacements in all directions are allowed. Among other findings, both periodic and non-periodic responses are reported.

Considering that the ultimate objective of the series of papers being developed by the authors in this theme is the derivation of mathematical models for seawater intake risers (SWIRs), a model of a clamped-free pipe is developed. In the following section, the model is derived and, after that, numerical simulations are discussed for four of the scenarios originally proposed for the planar model, in order to understand the simultaneous effect of internal and external flow to the response of the pipe. Finally, conclusions are drawn.

MODELING

The derivation of the model of a submerged cantilevered flexible pipe ejecting fluid under VIV is done under the following hypotheses:

(a) The pipe is inextensible, homogeneous and satisfies the hypotheses of linear elasticity.
(b) The motion in each cross section of the pipe can be adequately described using the coordinates of its center only.
(c) The internal flow is assumed to be a homogeneous plug flow with a relative velocity of constant magnitude V with respect to the pipe.
(d) The free-stream velocity associated to the external flow is assumed to be uniform and constant (both in magnitude U and direction, which is assumed to be horizontal).
(e) The interaction between the pipe and the external flow is modeled according to a two-dimensional wake-oscillator model which is assumed to be valid for each cross section of the pipe. This phenomenological model, originally applied for modeling VIV in a 2 degree-of-freedom (2-dof) rigid cylinder, was proposed by [14], under similar assumptions that led to the 1-dof model by [12]. The decoupling between cross sections in the phenomenological model is proposed under the assumption of a weak three-dimensionality of the flow around the pipe, which is discussed in [19], for the case of a flow around a fixed cylinder.

Hamiltonian formulation

Similarly to presented in [3–5, 7] the derivation of the mathematical model for the pipe will follow from the application of McIver's extended form of Hamilton's principle [10]:

$$\delta \int_{t_1}^{t_2} L\,dt + \int_{t_1}^{t_2} (\delta \tilde{W}_\mathrm{e} + \delta \tilde{W}_\mathrm{m})\,dt = 0 \qquad (1)$$

- L: Lagrangian of the system – inertial, flexural and gravitational effects (including buoyancy).
- $\delta \tilde{W}_\mathrm{e}$: virtual work associated to the interaction with the external flow – lift, drag and added mass effects.
- $\delta \tilde{W}_\mathrm{m}$: integral of the flux of momentum due to the internal flow, computed in the outlet surface of the pipe.

Define a global orthonormal basis $(\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z)$ fixed with respect to an inertial reference frame, such that $\mathbf{n}_z$ corresponds to the downward vertical and $\mathbf{n}_x$ to the direction of the free-stream external flow velocity. For each cross section of the pipe, define a local orthonormal basis $(\hat{\mathbf{e}}_\mathrm{x}, \hat{\mathbf{e}}_\mathrm{y}, \hat{\mathbf{e}}_\mathrm{z})$, such that $\hat{\mathbf{e}}_\mathrm{z}$ is orthogonal to the plane of the cross section and $\hat{\mathbf{e}}_\mathrm{y}$ is mutually orthogonal to $\hat{\mathbf{e}}_\mathrm{z}$ and to the direction $\hat{\mathbf{n}}_x$ of the free-stream velocity. The reader should be aware that the use of italic indexes x, y and z refers to the global basis $(\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z)$ whereas the use of upright indexes x, y and z refers to a particular local basis $(\hat{\mathbf{e}}_\mathrm{x}, \hat{\mathbf{e}}_\mathrm{y}, \hat{\mathbf{e}}_\mathrm{z})$. Let ξ be a non-dimensional arc-length coordinate (actual arc-length coordinate divided by the total length) defined along the center line of the pipe, such that each value of $0 \leq \xi \leq 1$ corresponds to a cross section of the pipe. Particularly $\xi = 0$ is the clamped end and $\xi = 1$ the free end of the cantilevered pipe. Denoting by $\mathbf{r}$ the non-dimensional position vector of a given point in the center line of the pipe with respect to the center of the clamped end and adopting the prime notation for partial derivatives with respect to ξ, it can be stated that: $\hat{\mathbf{e}}_\mathrm{z} = \mathbf{r}'$, $\hat{\mathbf{e}}_\mathrm{y} = \hat{\mathbf{e}}_\mathrm{z} \times \hat{\mathbf{n}}_x / \|\hat{\mathbf{e}}_\mathrm{z} \times \hat{\mathbf{n}}_x\|$ and $\hat{\mathbf{e}}_\mathrm{x} = \hat{\mathbf{e}}_\mathrm{y} \times \hat{\mathbf{e}}_\mathrm{z}$.

Let ℓ, D and d be the total length, the external diameter and the internal diameter of the pipe, respectively; let m_p, m_i and m_d be the linear mass densities of the pipe, of the the internal fluid in the pipe and of the displaced fluid in the external media; let m_a be the added mass per unit length (associated to usual potential flow considerations); let EI be the flexural rigidity (bending stiffness) of the pipe and $\mathbf{g} = g\hat{\mathbf{n}}_z$ be the local acceleration of gravity. Inspired by the classical non-dimensionals adopted for the classical problem of a pipe conveying fluid [9, 10], and following the conventions introduced in the derivation of the planar counterpart of this problem [3, 4], adopt $m_\mathrm{d}\ell$, ℓ and $\ell^2\sqrt{m_\mathrm{d}/EI}$ respectively as scales for mass, length and time. Also, adopt the overdot notation for partial derivatives with respect to the non-dimensional time variable τ. Non-dimensional

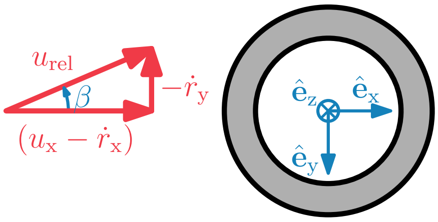

FIGURE 1: Components $\hat{\mathbf{e}}_{\rm x}$ and $\hat{\mathbf{e}}_{\rm y}$ of the relative velocity of the external flow with respect to (the center of) a cross section of the pipe. Adapted from [14].

expressions for L, $\delta W_{\rm e}$ and $\delta W_{\rm m}$ can be written in terms of the parameters defined in Table 1 as follows:

$$L = \frac{1}{2}\int_0^1 \left[(\mu_{\rm p}+\mu_{\rm i})\dot{\mathbf{r}}\cdot\dot{\mathbf{r}} + 2\mu_{\rm i}\nu\mathbf{r}'\cdot\dot{\mathbf{r}} + \mu_{\rm i}\nu^2\mathbf{r}'\cdot\mathbf{r}'\right]d\xi - \frac{1}{2}\int_0^1 \mathbf{r}''\cdot\mathbf{r}''d\xi + \gamma\int_0^1(\mu_{\rm p}+\mu_{\rm i}-1)z\,d\xi \quad (2)$$

$$\delta\tilde{W}_{\rm e} = \int_0^1\left[\frac{1}{2\pi\varepsilon}u^2(C_{\rm x}\hat{\mathbf{e}}_{\rm x}+C_{\rm y}\hat{\mathbf{e}}_{\rm y}) - \mu_{\rm a}\ddot{\mathbf{r}}\right]\cdot\delta\mathbf{r}\,d\xi \quad (3)$$

$$\delta\tilde{W}_{\rm m} = -\mu_{\rm i}\nu\left[(\dot{\mathbf{r}}+\nu\mathbf{r}')\cdot\delta\mathbf{r}\right]_{(\tau,1)} \quad (4)$$

$C_{\rm x}$ and $C_{\rm y}$ are the local force coefficients associated to the in-line and cross-wise directions of a cross section and are related to the drag and lift coefficients, $C_{\rm D}$ and $C_{\rm L}$ according to the following expressions:

$$\begin{cases} C_{\rm x} = (u_{\rm rel}/u)^2\left(C_{\rm D}\cos\beta - C_{\rm L}\sin\beta\right) \\ C_{\rm y} = (u_{\rm rel}/u)^2\left(C_{\rm D}\sin\beta + C_{\rm L}\cos\beta\right)\end{cases} \quad (5)$$

$u_{\rm rel}$ stands for the magnitude of the relative velocity of the external flow free-stream with respect to the center of a cross section and β to the associated angle of attack (see Fig. 1). It can be stated that $u_{\rm rel}^2 = (u_{\rm x}-\dot{r}_{\rm x})^2 + \dot{r}_{\rm y}^2$, $\cos\beta = (u_{\rm x}-\dot{r}_{\rm x})/u_{\rm rel}$ and $\sin\beta = -\dot{r}_{\rm y}/u_{\rm rel}$, with $u_{\rm x} = (u\hat{\mathbf{n}}_x)\cdot\hat{\mathbf{e}}_{\rm x}$, $\dot{r}_{\rm x} = \dot{\mathbf{r}}\cdot\hat{\mathbf{e}}_{\rm x}$ and $\dot{r}_{\rm y} = \dot{\mathbf{r}}\cdot\hat{\mathbf{e}}_{\rm y}$.

Particularly, according to the two-dimensional wake-oscillator model adopted [14], the drag and lift coefficients have oscillating parcels which are proportional to the in-line ($q_{\rm x}$) and cross-wise ($q_{\rm y}$) wake-variables, respectively, that is: $C_{\rm D} = \bar{C}_{\rm D}^0 + (q_{\rm x}/\hat{q})\hat{C}_{\rm D}^0$ and $C_{\rm L} = (q_{\rm y}/\hat{q})\hat{C}_{\rm L}^0$. Definitions and values of $\bar{C}_{\rm D}^0$, $\hat{C}_{\rm D}^0$, $\hat{C}_{\rm L}^0$ and $\hat{q}$ definitions are presented in Tab. 1. The wake-variables dynamics follow by the following van der Pol equations forced by terms proportional to the accelerations of the pipe in the respective directions (see Tab. 1 for the definition of the parameters):

$$\begin{cases} \ddot{q}_{\rm x} + \varepsilon_{\rm x}\tilde{\omega}_{\rm s}(q_{\rm x}^2-1)\dot{q}_{\rm x} + (2\tilde{\omega}_{\rm s})^2 q_{\rm x} = (A_{\rm x}/\varepsilon)\ddot{\mathbf{r}}\cdot\hat{\mathbf{e}}_{\rm x} \\ \ddot{q}_{\rm y} + \varepsilon_{\rm y}\tilde{\omega}_{\rm s}(q_{\rm y}^2-1)\dot{q}_{\rm y} + \tilde{\omega}_{\rm s}^2 q_{\rm y} = (A_{\rm y}/\varepsilon)\ddot{\mathbf{r}}\cdot\hat{\mathbf{e}}_{\rm y}\end{cases} \quad (6)$$

The non-dimensional vortex-shedding angular frequency is $\tilde{\omega}_{\rm s} = 2\pi u St/\varepsilon$, with St being the Strouhal number. A lock-in peak occurs when $\tilde{\omega}_{\rm s} = |\tilde{\lambda}_n|$, being $\tilde{\lambda}_n$ the eigenvalue associated to the n-th mode of the non-dimensional model of a pipe ejecting fluid in the absence of any dynamic effects due to the external media. Therefore, the value of u corresponding to the n-th mode lock-in peak is $u = \varepsilon\tilde{f}_n/St$, with $\tilde{f}_n = |\tilde{\lambda}_n|/2\pi$. Indeed, in terms of the non-dimensional free-stream velocity typically adopted in the literature for VIV, which is defined as $U^* = U/(Df_n) = u/(\varepsilon\tilde{f}_n)$, the n-th mode lock-in peak corresponds to the condition $U^* = 1/St$. In the case of 2-dof VIV [14], the value of U^* associated to the peak of amplitude of oscillations in cross-wise direction is higher than in 1-dof VIV leading to $St = 0.17$.

Derivation of a reduced order model

Define $\mathbf{h}(\xi)$ as a column-matrix of projection functions satisfing the boundary conditions $\mathbf{h}(0) = \mathbf{h}'(0) = \mathbf{h}''(1) = \mathbf{h}'''(1) = 0$. As discussed in [3, 4], for lock-in scenarios, the oscillations of the wake are dominated by the vibrations of natural modes of the structure so that the same set of projection functions can be adopted for the discretization of both the Cartesian coordinates (x,y,z) of points along the center line of the pipe and the wake-variables $q_{\rm x}$ and $q_{\rm y}$. This leads to the following Galerkin scheme: $x(\tau,\xi) = \mathbf{h}(\xi)\cdot\mathbf{r}_x(\tau)$, $y(\tau,\xi) = \mathbf{h}(\xi)\cdot\mathbf{r}_y(\tau)$, $z(\tau,\xi) = \xi + \mathbf{h}(\xi)\cdot\mathbf{r}_z(\tau)$, $q_{\rm x}(\tau,\xi) = \mathbf{h}(\xi)\cdot\mathbf{q}_{\rm x}(\tau)$ and $q_{\rm y}(\tau,\xi) = \mathbf{h}(\xi)\cdot\mathbf{q}_{\rm y}(\tau)$. Also following [3, 4], all the non-linear terms of the model are replaced by the following redundant variables so that the discretization of the relaxed model becomes a straighforward procedure ($j = x,y,z$ and ${\rm k} = {\rm x},{\rm y}$):

$$\begin{cases} \dot{c}_j(\tau,\xi) = (C_{\rm x}\hat{\mathbf{e}}_{\rm x}+C_{\rm y}\hat{\mathbf{e}}_{\rm y})\cdot\hat{\mathbf{n}}_j = \mathbf{h}(\xi)\cdot\dot{\mathbf{c}}_j(\tau) \\ \dot{p}_{\rm k}(\tau,\xi) = (q_{\rm k}^2-1)\dot{q}_{\rm k} = \mathbf{h}(\xi)\cdot\dot{\mathbf{p}}_{\rm k}(\tau) \\ \ddot{a}_{\rm k}(\tau,\xi) = \ddot{\mathbf{r}}\cdot\hat{\mathbf{e}}_{\rm k} = \mathbf{h}(\xi)\cdot\ddot{\mathbf{a}}_{\rm k}(\tau) \\ e'(\tau,\xi) = \sqrt{y'^2+z'^2} = 1 + \mathbf{h}'(\xi)\cdot\mathbf{e}(\tau) \\ \dot{w}(\tau,\xi) = u_{\rm rel} = \mathbf{h}(\xi)\cdot\dot{\mathbf{w}}(\tau)\end{cases} \quad (7)$$

Thus, the dynamics of the relaxed model can be expressed by the following system of linear equations ($j = x,y,z$ and ${\rm k} = {\rm x},{\rm y}$):

$$\begin{cases} (\mu_{\rm p}+\mu_{\rm i}+\mu_{\rm a})\mathbf{H}^{00}\ddot{\mathbf{r}}_j + \mu_{\rm i}\nu(\mathbf{H}^{01}-\mathbf{H}^{10}+\mathbf{E}^{00})\dot{\mathbf{r}}_j \\ \quad + \left(\mathbf{H}^{22}+\mu_{\rm i}\nu^2(\mathbf{E}^{01}-\mathbf{H}^{11})\right)\mathbf{r}_j = \dfrac{u^2}{2\pi\varepsilon}\mathbf{H}^{00}\dot{\mathbf{c}}_j \\ \quad + \alpha_j\gamma(\mu_{\rm p}+\mu_{\rm i}-1)\mathbf{h}^0 \\ \ddot{\mathbf{q}}_{\rm k} + \varepsilon_{\rm k}\dfrac{2\pi u St}{\varepsilon}\dot{\mathbf{p}}_{\rm k} + \left(\rho_{\rm k}\dfrac{2\pi u St}{\varepsilon}\right)^2\mathbf{q}_{\rm k} = \dfrac{A_{\rm k}}{\varepsilon}\ddot{\mathbf{a}}_{\rm k}\end{cases} \quad (8)$$

with $\alpha_x = \alpha_y = 0$, $\alpha_z = 1$, $\rho_{\rm x} = 2$ and $\rho_{\rm y} = 1$.

$\mathbf{H}^{ij}$ and $\mathbf{h}^i$ correspond to the typical coefficient matrices of linear weak-formulations of one-dimensional continuous systems, while $\mathbf{E}^{ij}$ is associated to the discretization of the terms

TABLE 1: Non-dimensional parameters of the model and the values adopted for them in the numerical simulation scenarios – parameters from the in-line and cross-wise wake-oscillator models are taken from [14].

Parameter	Description	Definition	Value
u	Non-dimensional free-stream velocity of the external flow	$u = U\ell\sqrt{m_\mathrm{d}/EI}$	Tab. 2
v	Non-dimensional relative velocity of the internal plug flow	$v = V\ell\sqrt{m_\mathrm{d}/EI}$	Tab. 2
ε	Slenderness ratio of the pipe	$\varepsilon = D/\ell$	0.02
γ	Non-dimensional geometric rigidity of the pipe	$\gamma = g\ell^3 m_\mathrm{d}/EI$	0
μ_p	Pipe to displaced fluid linear density ratio	$\mu_\mathrm{p} = m_\mathrm{p}/m_\mathrm{d}$	1.92
μ_i	Internal to displaced fluid linear density ratio	$\mu_\mathrm{i} = m_\mathrm{i}/m_\mathrm{d}$	0.48
μ_a	Added mass coefficient (potential flow theory)	$\mu_\mathrm{a} = m_\mathrm{a}/m_\mathrm{d}$	1
$\bar{C}^0_\mathrm{D}$	Mean drag coefficient of a stationary cylinder		1.1856
$\hat{C}^0_\mathrm{D}$	Amplitude of the drag coefficient of a stationary cylinder		0.2
$\hat{C}^0_\mathrm{L}$	Amplitude of the lift coefficient of a stationary cylinder		0.3842
$\hat{q}$	Limit-cycle amplitude of an unforced van der Pol oscillator		2
ε_x	In-line wake-oscillator van der Pol parameter		0.6
ε_y	Cross-wise wake-oscillator van der Pol parameter		0.00778
A_x	In-line acceleration coupling coefficient		12
A_y	Cross-wise acceleration coupling coefficient		2
St	Strouhal number		0.17

of $\delta\tilde{W}_\mathrm{m}$, computed at $\xi = 1$. Letting $\otimes$ denote the outer product and adopting $\mathbf{h}^{(0)} = \mathbf{h}$, $\mathbf{h}^{(1)} = \mathbf{h}'$ and $\mathbf{h}^{(2)} = \mathbf{h}''$, it can be stated that: $\mathbf{H}^{ij} = \int_0^1 \mathbf{h}^{(i)} \otimes \mathbf{h}^{(j)}\, d\xi$, $\mathbf{h}^i = \int_0^1 \mathbf{h}^{(i)}\, d\xi$ and $\mathbf{E}^{ij} = \left[\mathbf{h}^{(i)} \otimes \mathbf{h}^{(j)}\right]\big|_{\xi=1}$.

Noticing that the proposed discretization scheme identically satisfies all the boundary conditions of the system, the constraints that must be enforced *a posteriori* are the ones given by Eqs. (7), related to the definition of redundant variables, and the ones related to the inextensibility condition $x'^2 + y'^2 + z'^2 = 1$. All these constraints are enforced according to the Modular Modeling Methodology [1,2].

In the discretized problem, assume that $\mathbf{h} \in \mathbb{R}^n$. The enforcement of these constraints can only be ensured at n points along the structure (for instance at the points $\xi_k = k/n$, $k = 1,\ldots,n$). Such a limitation, however, does not seem to affect significantly the precision of the results, as illustrated in [7] by a comparison between a reduced order model obtained under the same assumptions and a non-linear FEM model of a planar cantilevered pipe ejecting fluid.

NUMERICAL SIMULATIONS AND DISCUSSIONS

In order to be able to compare the response of the model derived in this paper with its planar counterpart, previously assessed in [3, 4], in which the motion of the pipe is artificially constrained to a vertical plane (orthogonal to the direction of the external free-stream), the same numerical simulations scenarios are proposed. As it can be noticed in Tab. 2, three conditions for the internal flow are proposed, based on the response of the pipe as a function of v. For the chosen set of parameters (see Tab. 1), increasing the value of v (starting from $v = 0$), the first observable bifurcation occurs at the second mode, so that $v = 7$ and $v = 7.35$ correspond to subcritical (stable) and supercritical (unstable) scenarios, respectively [3,4]. In scenarios E2, S2 and U2 the values of v are 0, 7 and 7.35, respectively, and the values of u are tuned to the peak of the second mode lock-in, i.e., $u = \varepsilon\tilde{f}_2/St$, with $\tilde{f}_2$ standing for the frequency of the second mode of the pipe ejecting fluid. In scenario U0, the supercritical response of the system is assessed in the absence of an external free-stream ($u = 0$). In this latter case, the interaction of the pipe with the external fluid is restricted to the effects of buoyancy, added mass and drag (with a constant drag coefficient, equal to $\bar{C}^0_\mathrm{D}$).

The simulation results shown in Figs. 2 and 3 show the steady responses of the corresponding scenarios. The plotted curves correspond to selected positions along the center line of the pipe $\xi = 0.2, 0.4, 0.6, 0.8, 1$. x/ε and y/ε are the renormalized forms of the non-dimensional Cartesian coordinates with respect to the diameter of the pipe, which is the length scale usually applied in VIV analyses. In Fig. 2 the simula-

TABLE 2: Numerical simulation scenarios.

Scenario	External flow	Internal flow
E2	Tuned to the peak of 2nd mode lock-in	$v = 0$: internal flow absent
S2	Tuned to the peak of 2nd mode lock-in	$v = 7$: subcritical (below 2nd mode bifurcation of the pipe)
U2	Tuned to the peak of 2nd mode lock-in	$v = 7.35$: supercritical (above 2nd mode bifurcation of the pipe)
U0	$u = 0$	$v = 7.35$: supercritical (above 2nd mode bifurcation of the pipe)

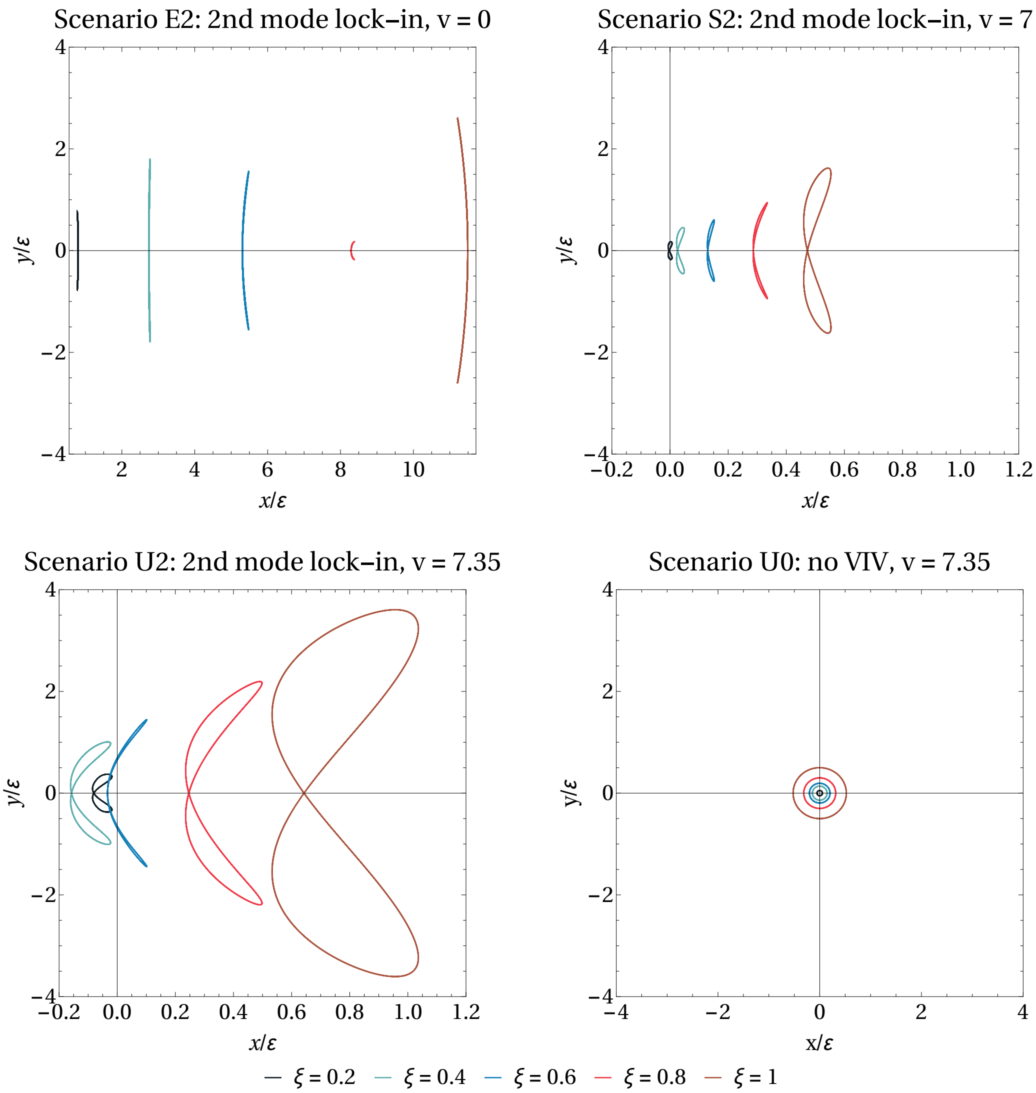

FIGURE 2: y/ε vs. x/ε plots for selected points along the pipe associated to the scenarios E2, S2, U2 and U0, in which 2nd mode oscillations dominate the dynamic response.

tion results reproduces qualitatively the expected response: for scenarios E2, S2 and U2, which correspond to the peak of second mode lock-in the in-line oscillations occur with twice the frequency observed in the cross-wise ones. Also, drag effect is noticeable in the in-line direction. As already noticed in the planar case [3, 4], the stability of the pipe ejecting fluid plays a fundamental role in the amplitude of the cross-wise oscillations: in the subcritical scenario S2 there is a mitigation of this

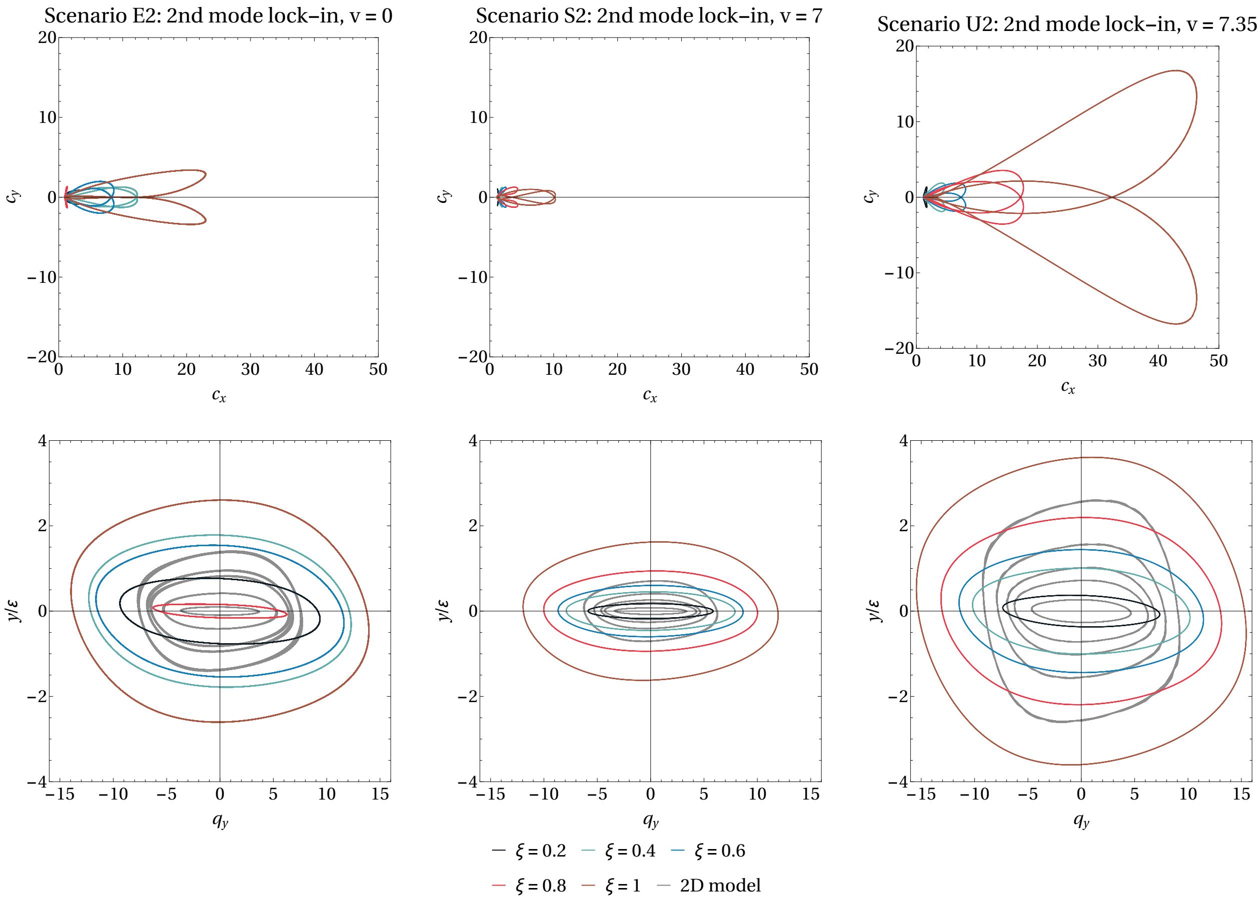

FIGURE 3: c_y vs. c_x and y/ε vs. q_y plots for selected points along the pipe associated to the scenarios E2, S2 and U2 in which 2nd mode oscillations dominate the dynamic response. The corresponding y/ε vs. q_y plots obtained for the same scenarios, from the simulation of a planar model [3,4] are also shown in gray, for comparison.

motion (taking E2 as reference), while in the supercritical scenario U2 the amplitudes are amplified. Considering that the values of u adopted in scenarios E2, S2 and U2 are different (due to the dependency of $\tilde{f}_2$ on v), a less distorted comparison concerning the influence of the stability of the pipe ejecting fluid can be done by assessing the c_y vs. c_x curves presented in Fig. 3. These are the force coefficients associated to the directions $\hat{\mathbf{n}}_y$ and $\hat{\mathbf{n}}_x$, respectively, and the mitigation effect in the subcritical scenario S2 and the amplification in the supercritical scenario U2, when compared to E2, are noticeable. Another relevant comparison that can be performed with the planar model, consists in analyzing the y/ε vs. q_y plots. As expected, in the lock-in peaks there is a common dominant frequency between the cross-wise motion of the pipe and the cross-wise wake variable oscillations. However, in all three analyzed scenarios, for all the selected positions along the center line, the three-dimensionality of the motion of the pipe leads to larger amplitudes of oscillations for both of these variables, as also shown in Fig. 3.

CONCLUSIONS

Even under several simplifying hypotheses that still require experimental verification, the non-linear reduced order model derived in this paper seems to reproduce, at least qualitatively, the dynamic response that would be expected for the selected simulation scenarios, particularly on the influence of the stability of the pipe due to internal flow in its oscillations under VIV. The existence of a range of internal flow speeds for which the amplitude of oscillations is mitigated can be properly explored in the design of riser systems.

From a qualitative point of view, the results obtained are similar to the ones already discussed for the planar model [3,4], with the noticeable difference that when the pipe is also allowed to oscillate in the in-line direction, the cross-wise amplitudes are amplified, what is usually true in the case of VIV of rigid cylinders as well.

Future work will focus on a detailed analysis of the non-linear response of this model, including the construction of stability maps having u and v as control parameters. Also, the influence of the choice of projection functions in the Galerkin scheme shall be further discussed.

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