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# NONLINEAR RESPONSE OF PIPES WITH A BREATHING CRACK UNDER HARMONIC LOAD

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## ABSTRACT

In this paper, the formula of local flexibility at crack location in pipes was established. According to the compatibility conditions of displacement, rotation angle, shear force and bending moment and the boundary conditions of the cracked pipes, the mode shape functions of the cracked pipes were established. Then based on the Lagrange's equation, an equation of motion for pinned-pinned pipes with a breathing crack under harmonic load were derived. The equations of motion for the cracked pipe were discretized by the modal functions of cracked pipe, and the discrete equations for the cracked pipe were obtained. The mass matrix, damping matrix, stiffness matrix varied with time due to opening and closing of the crack. Then numerical methods were used to solve the discrete equations and the dynamic response characteristics of the cracked pipe conveying fluid under harmonic excitation load were studied. The super-harmonic resonance and sub-harmonic resonance could be shown when the pipes was under harmonic excitation load. In this paper the influence of system parameters on the harmonic components in the system response were investigated finally, such as crack depth. And a ratio is used to describe the relative magnitude of each harmonic component in the response of cracked pipes. These results could provide a reference for crack identification for cracked pipes.

## 1.INTRODUCTION

The presence of cracks would affect the mass, damping and stiffness characteristics of a structure, and these properties change with the opening and closing states of the cracks, and these changes further affect the structural response.

Many researchers have begun to pay attention to the non-linear characteristics of the structure due to the crack opening and closing. Shen et al[1] used the spring-mass oscillator model to simulate the modal vibrations of various modes of supported beams with a breathing cracked. Andreaus et al.[2] Hu et al.[3]and CHEN YAN[4] using two-dimensional finite element method to establish The cracked model. The friction-free contact element was used to simulate the crack opening and closing. The super-harmonic resonance and sub-harmonic resonance of the cracked cantilever under simple harmonic excitation. Giannini et al.[5] Studied sub-harmonic and super-harmonic resonances of cracked beams under simple harmonic excitation. The concept of harmonic damage surface (HDS) is proposed, and the crack parameters are identified by the ratio of the amplitude corresponding to the excitation frequency and the amplitude corresponding to the sub-harmonic / super resonance under harmonic excitation

Yoon et al.[6] studied the dynamic behavior of a cracked cantilever pipe conveying flow and a simply supported pipe conveying flow under moving loads. Cai Fengchun et al.[7-9] discussed the stability of cracked cantilever pipe conveying flow, and discussed the effect of crack location and depth on frequency and flutter critical flow rate, and studied the nonlinear dynamic characteristics of the pinned-pinned pipe under pulsating flow. Bao Ridong et al. [10] studied the dynamica characteristics of a cracked pipe conveying fluid with both elastically supported ends. Results showed that the natural frequencies and the fluid critical flow velocity vary complicatedly due to crack existing.

In this paper, modal function of cracked beam was established by transverse bending vibration mode function of beam model which was added by a cubic polynomial. Based on the Lagrange equation, the equations of motion of the pinned-pinned pipe with cracks under harmonic excitation were established, and the super-harmonic resonance and sub-harmonic resonance of cracked pipe under harmonic excitation are studied.

## 2. LOCAL COMPLIANCE COEFFICIENT OF CRACK

Assuming that the crack tip stress is within the elastic range, The crack region with uniaxial straight cracked pipe is divided into a series of infinite rectangular strip regions (see Fig. 1). The stress intensity factor of each tiny cracked strip is near the single crack stress intensity factor of the infinite strip, Then obtain the compliance coefficient of straight crack.

Stress intensity factor at the strip[11]:

$$K_I = \frac{M_c \tilde{h}}{2I}\sqrt{\pi\tilde{\xi}}F(\tilde{x}) \quad (1)$$

where , $\tilde{\xi} = \sqrt{(D_e/2)^2 - (\tilde{r}\sin\tilde{\theta})^2} - \tilde{r}\cos\tilde{\theta}$ , $\tilde{h} = 2\sqrt{(D_e/2)^2 - (\tilde{r}\sin\tilde{\theta})^2}$ , $\tilde{x} = \tilde{\xi}/\tilde{h}$ , $M_c$ is bend moment , $I = \pi D_e^4(1-\gamma^4)/64$ , $\gamma = D_i/D_e$ , $a$ is crack depth, $h$ is Pipe wall thickness.

$$F(\tilde{x}) = \sqrt{2/\pi\tilde{x}\tan(\pi\tilde{x}/2)}\{0.923 \\ +0.199[1-\sin(\pi\tilde{x}/2)]^4\}/\cos(\pi\tilde{x}/2)$$

According to the theory of fracture mechanics, Based on castigliano's theorem, the local compliance coefficient caused by the circumferential crack in the outer wall under the action of pure bending moment can be calculated as:

$$C = \frac{\partial^2}{\partial M_c^2}\int_{A_c}\frac{K_I^2}{E'}dA \\ = \frac{2048}{\pi E' D_e^3(1-\gamma^4)^2}\int_0^{\theta}\int_{D_e/2-a}^{D_e/2}\{[D_e^2 - 4(\tilde{r}\sin\tilde{\theta})^2] \\ [\sqrt{D_e^2 - 4(\tilde{r}\sin\tilde{\theta})^2} - 2\tilde{r}\cos\tilde{\theta}][F(\tilde{x})]^2\}\tilde{r}d\tilde{r}d\tilde{\theta} \quad (2)$$

where , $E' = E/(1-\nu^2)$ , $E$ is Elastic Modulus , $\nu$ is Poisson's ratio , $A_c$ is crack area

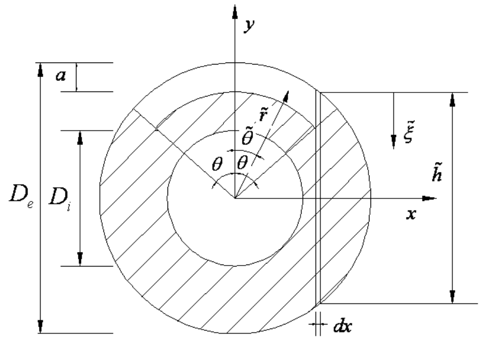



**Figure** 1 partly circumferential crack

## 3. MODAL FUNCTION FOR CRACKED BEAM

Beam lateral displacement can be written as:

$$y(X,t) = \sum_{j=1}^{n} q_j(t)\phi_j(X) = \mathbf{\Phi(X)q(t)} \quad (3)$$

where , $\mathbf{\Phi(X)} = [\phi_1(X)\ \phi_2(X) \cdots \phi_n(X)]$

$\mathbf{q(t)} = [q_1(t)\ q_1(t) \cdots q_n(t)]^T$

Considering a beam containing a crack, the crack beam is divided into two sections, each assembled with a massless torsion springs. The j-th modal function of cracked beam can be expressed by piecewise functions:

$$\begin{cases}\phi_{j1}(\xi)=\bar{\phi}_j(\xi)+A_1+A_2\xi+A_3\xi^2+A_4\xi^3 & 0\le\xi\le\xi_c\\ \phi_{j2}(\xi)=\bar{\phi}_j(\xi)+A_5+A_6\xi+A_7\xi^2+A_8\xi^3 & \xi_c<\xi\le 1\end{cases} \tag{4}$$

where , $\xi = X/L$ , $\xi_c = X_c/L$ , is the length of beam , $A_1, A_2, \cdots, A_8$ is coefficient, $\bar{\phi}_j(\xi)$ is Non-cracked beam modal function.

For a pinned- pinned beams, there are 4 boundary conditions:

$$\phi_{j1}(0)=0 \tag{5}$$

$$\phi''_{j1}(0)=0 \tag{6}$$

$$\phi_{j1}(1)=0 \tag{7}$$

$$\phi''_{j1}(1)=0 \tag{8}$$

At crack location, translational displacement, angular displacement, shear and moment should satisfy the following four coordination conditions:

$$\phi_{j1}(\xi_c)=\phi_{j2}(\xi_c) \tag{9}$$

$$\phi''_{j1}(\xi_c)=\phi''_{j2}(\xi_c) \tag{10}$$

$$\phi'''_{j1}(\xi_c)=\phi'''_{j2}(\xi_c) \tag{11}$$

$$\phi'_{j2}(\xi_c)-\phi'_{j1}(\xi_c)=EIC\phi''_{j2}(\xi_c)/L \tag{12}$$

where , $C$ is local compliance coefficient of crack , $(')=\partial/\partial\xi$ .

The modal function of a cracked beam can be uniquely determined by Eqs. (5) ~ (12).

## 4. EQUATION OF MOTION

pinned-pinned cracked pipe model is shown in Fig. 2. The pipe length is $L$ . Pipe cross-sectional area $A_p$ . The mass of the unit length of pipe $m$ . Bending stiffness is $EI$ . Crack location is $X_c$ . The beam is placed along the axis $X$ . Harmonic excitation $F_0^*\sin\omega^* t$ is at $X_f$ .

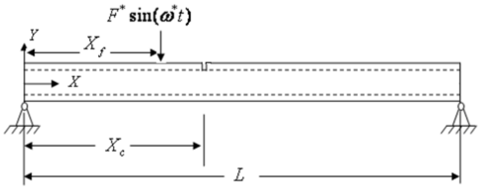



**Figure 2** A pinned-pinned cracked beam with hollow section

Kinetic energy of pipe is

$$T=\frac{1}{2}m\int_0^L \dot{y}^2 dX=\frac{1}{2}m\int_0^L \dot{\mathbf{q}}^{\mathrm{T}}\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}\dot{\mathbf{q}}dX \tag{13}$$

The elastic potential energy caused by bending deformation is:

$$V_b=\frac{1}{2}\int_0^L EIy''^2 dX \tag{14}$$

The potential energy of a massless spring used to simulate a crack is:

$$V_c=\frac{1}{2}\frac{1}{C}[y'_{X_c^+}-y'_{X_c^-}]^2=\frac{1}{2}C[EIy''_{X_c}]^2 \tag{15}$$

External load work:

$$V_f=-F_0^*\sin\omega^* t\int_0^L y\delta(X-X_f)dX \tag{16}$$

The system kinetic energy and potential energy are substituted into the Lagrange equation to obtain the equation of motion of pipes with a open crack:

$$\begin{aligned}&m\int_0^L \ddot{y}\frac{\partial\dot{y}}{\partial\dot{q}_i}dX+\int_0^L C^*\dot{y}\frac{\partial y}{\partial q_i}dX\\&+\int_0^L EIy''\frac{\partial y''}{\partial q_i}dX+C(EI)^2 y''_{X_c}\frac{\partial y''_{X_c}}{\partial q_i}\\&=F_0^*\sin\omega^* t\frac{\partial y_{X_f}}{\partial q_i}\end{aligned} \tag{17}$$

In order to obtain the dimensionless equations of motion, the following dimensionless quantities are introduced:

$\boldsymbol{\eta}=\frac{\mathbf{q}}{L}$ , $\xi=\frac{X}{L}$ , $\tau=(\frac{EI}{m})^{1/2}\frac{t}{L^2}=\omega_n t$ ,
$k_c=CEI/L$ , $\xi_c=X_c/L$ , $\Omega=\omega^*/\omega_n$ ,
$f_0^*=F_0^*L^2/EI$ , $\xi_f=X_f/L$ ,
$c=C^*L^2/\sqrt{EIm}$ ,

Substituting Eq. (3) into Eq. (17), the mass, damping, stiffness matrix and load vector of the system can be obtained.

$$\begin{cases} \tilde{\mathbf{M}}_{\mathbf{o}} = \mathbf{S}_{00} \\ \tilde{\mathbf{C}}_{\mathbf{o}} = c\mathbf{S}_{00} \\ \tilde{\mathbf{K}}_{\mathbf{o}} = \mathbf{S}_{22} + k_c\tilde{\mathbf{S}}_{22} \\ \tilde{\mathbf{f}}_{\mathbf{o}} = \mathbf{f}_{\mathbf{0}}^{*} \sin\Omega\tau\tilde{\mathbf{S}}_{\mathbf{0}} \end{cases} \quad (18)$$

where , $\mathbf{S}_{00} = \int_0^1 \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi} d\xi$ , $\mathbf{S}_{11} = \int_0^1 \mathbf{\Phi}'^{\mathrm{T}}\mathbf{\Phi}' d\xi$ , $\mathbf{S}_{22} = \int_0^1 \mathbf{\Phi}''^{\mathrm{T}}\mathbf{\Phi}'' d\xi$ , $\tilde{\mathbf{S}}_{22}(\xi_c) = \mathbf{\Phi}''^{T}(\xi_c)\mathbf{\Phi}''(\xi_c)$ , $\tilde{\mathbf{S}}_{\mathbf{0}} = \mathbf{\Phi}^{\mathrm{T}}(\xi_f)$ , $(') = \partial / \partial\xi$ 。

When the crack is closed, the local compliance coefficient $C$ is zeros. In this case, the equation of motion for the cracked pipe is the same as the equation of motion for the uncracked pipe.Based on the formula (18), when the crack is closed, the system mass matrix $\tilde{\mathbf{M}}_{\mathbf{c}}$ , damping matrix $\tilde{\mathbf{C}}_{\mathbf{c}}$ , stiffness matrix $\tilde{\mathbf{K}}_{\mathbf{c}}$ and load column vector $\tilde{\mathbf{f}}_{\mathbf{c}}$ can be obtained.

The equation of motion for a pinned-pinned pipe with breathing crack can be written as:

$$\begin{cases} \tilde{\mathbf{M}}_{\mathbf{o}}\ddot{\mathbf{\eta}} + \tilde{\mathbf{C}}_{\mathbf{o}}\dot{\mathbf{\eta}} + \tilde{\mathbf{K}}_{\mathbf{o}}\mathbf{\eta} = \tilde{\mathbf{f}}_{\mathbf{o}} & \mathbf{\Phi}''(\xi_c)\eta(\tau) > 0 \\ \tilde{\mathbf{M}}_{\mathbf{c}}\ddot{\mathbf{\eta}} + \tilde{\mathbf{C}}_{\mathbf{c}}\dot{\mathbf{\eta}} + \tilde{\mathbf{K}}_{\mathbf{c}}\mathbf{\eta} = \tilde{\mathbf{f}}_{\mathbf{c}} & \mathbf{\Phi}''(\xi_c)\eta(\tau) \le 0 \end{cases} \quad (19)$$

where , $(') = \partial / \partial\xi$ , $(^{\bullet}) = \partial / \partial\tau$ , $\mathbf{\Phi}''(\xi_c)\eta(\tau)$ is the curvature of pipe at the crack position 。

## 5. NUMERICAL SIMULATION

In this section, Newmark numerical integration method is used to calculate the dynamic response of the system. The steady-state acceleration response at the middle point of the pipe is taken and the acceleration power spectral density(PSD) is calculated to analyze the frequency components in the acceleration response.

The parameter is taken as $L = 1m$ , $E = 2.0 \times 10^{11} Pa$ , $\nu = 0.3$ , $D_e = 0.03m$ , $D_i = 0.021m$ , $c = 0.1$ , $f_0^* = 0.1$ , $\xi_f = 0.5$ , $a / h = 0.8$ , $\theta = \pi / 4$ , $\xi_c = 0.5$

The first order bilinear dimensionless frequency can be defined as:

$$\omega_b^1 = 2\omega_c^1\omega_o^1 / (\omega_c^1 + \omega_o^1) \quad (20)$$

where, $\omega_o^1$ is the first order dimensionless frequency of the system when the crack is open, $\omega_c^1$ is the first order dimensionless frequency of the system when the crack is close.

A ratio of dimensionless frequency is defined as:

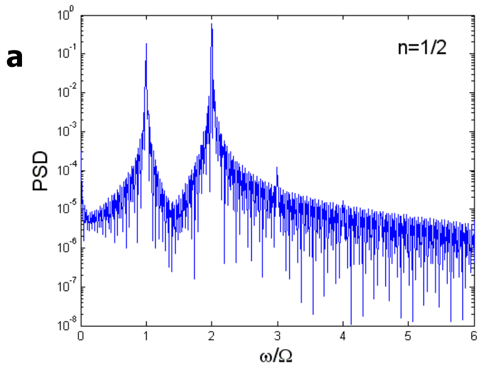
$$n = \Omega / \omega_b^1$$

where, $\Omega$ is dimensionless frequency of harmonic force.

### 5.1 EFFECT OF EXCITATION FREQUENCY

In order to study the superharmonic resonance and subharmonic resonance of a pipe with a breathing crack under harmonic excitation. The excitation frequency is taken as 1/4, 1/3, 1/2,twice of the first order bilinear frequency $\omega_b^1$ , that is $n = 1/4, 1/3, 1/2, 2$ . The results under different excitation frequency are shown in Figure 3a, 3b, 3c, 3d.

Due to the nonlinearity of the crack, sub-harmonic resonance or sup-erharmonic resonance obviously occurs when the excitation frequency is an integral multiple or a fractional division of the first order bilinear frequency.The response produced multiple harmonic components. The energy of each harmonic component is different. And the proportion is not the same. Particularly , the amplitude of the harmonic components near the first-order bilinear frequency $\omega_b^1$ is more significant than that of other harmonic components.



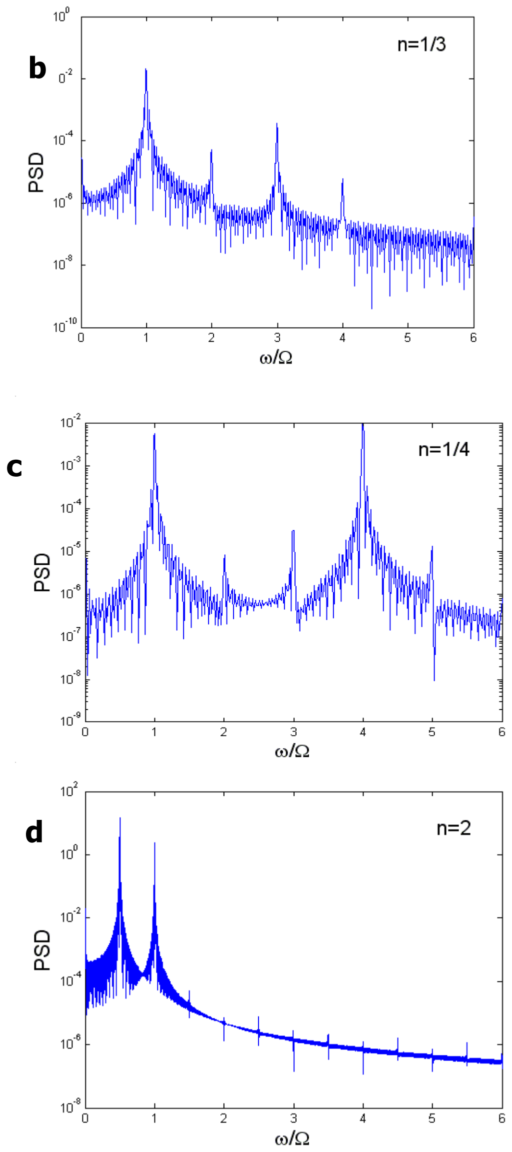



**Figure** 3 acceleration response of pipes at different excitation frequencies

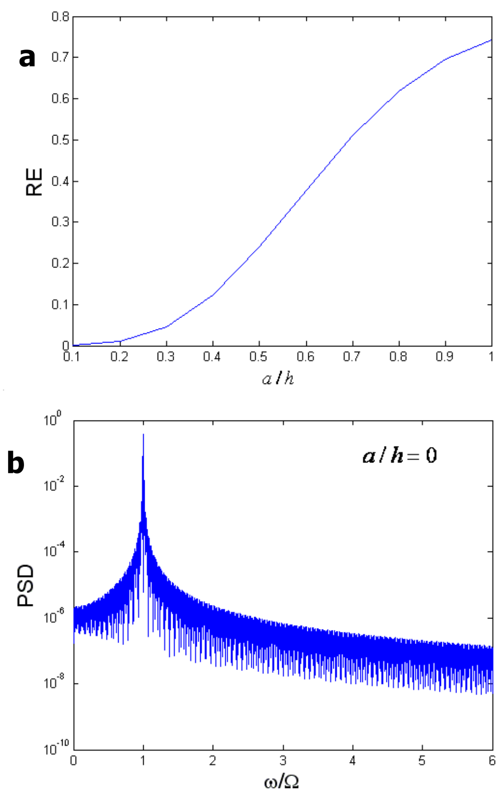
### 5.2 EFFECT OF CRACK DEPTH

From the results of the section 4.1, it can be seen that the amplitude of the harmonic component corresponding to the first order bilinear frequency is very high when n is 1/2. The energy of system response is mainly concentrated on 2 harmonic components. In this section, we will define the ratio of the energy of second harmonic components to all the response energy to depict the change of the energy of 2 harmonic components.

$$RE = \frac{E_2}{E_1 + E_2} \qquad (21)$$

where, $E_1$ is root mean square value of first harmonic components, $E_2$ is root mean square value of second harmonic components.

The crack depth is changed(crack position $\xi_c = 0.5$), and the curve of the RE value with the crack depth is calculated. Results shown in fig.4a, it can be seen that the RE value increases monotonously with the increase of the crack depth. When the depth of the crack is taken as $a/h = 0, 0.3, 0.6$, the results are shown in fig.4b, fig.4c, and fig.4d. It can be seen that when the depth of the crack is zero, the system is a nondestructive pipe, and the response energy of the system is all concentrated at the excitation frequency.When a crack appears, the harmonic component at 2 times the frequency of the excitation frequency appears.With the increase of the depth of the crack, the ratio of the energy of the second harmonic components (the first order bilinear frequency) to the total response energy of the system is also increasing. Therefore, the RE value can be used to monitor the change of the depth of the crack in the pipe.



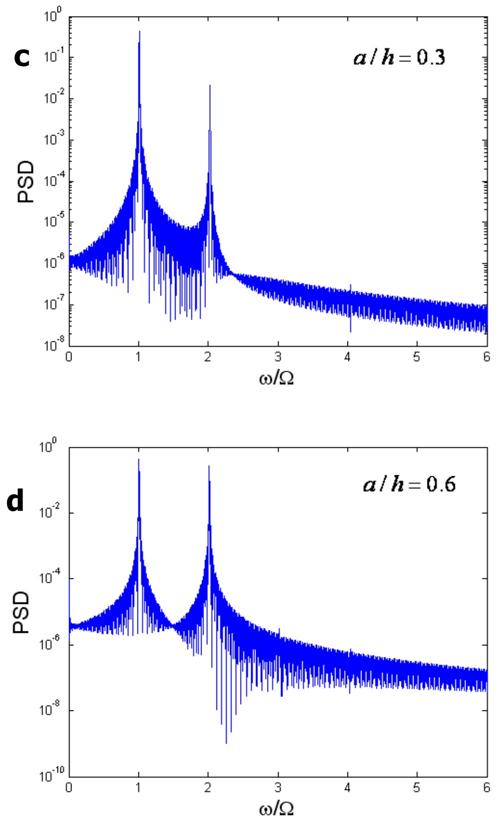



**Figure 4** Effect of crack depth

## 6. CONCLUSION

The sub-harmonic resonance and super-harmonic resonance of the breathing cracked pipe under harmonic excitation at different frequencies are analyzed in this paper. Due to the nonlinearity of breathing crack, many harmonic components in response are aroused when the excitation frequency is an integer multiple or a fraction of the first order bilinear frequency, and the amplitude of the harmonic component near the first order bilinear frequency is more significant than the other harmonic components. By using these characteristics, the relative ratio of the energy of the harmonic component can be defined, which can be used to monitor the change of the depth of the crack.

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